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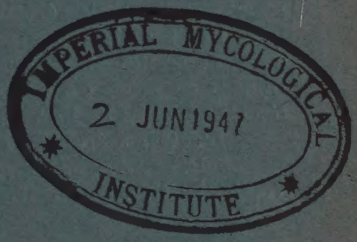

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MATHÉMATIQUES

SUR LES LIGNES SITUÉES SUR UNE SURFACE DONT LA TORSION GÉODÉSIQUE, LA COURBURE NORMALE ET LA COURBURE GÉODÉSIQUE SONT LIÉES PAR UNE RELATION LINÉAIRE À COEFFICIENTS CONSTANTS

Par N. TOUGANOFF

(Présenté par N. N. Lusin, de l'Académie, le 18. XI. 1940)

1. L'origine géométrique de la classe des lignes $A\frac{1}{T_g}+$ $+B\nu+C\frac{1}{\rho_g}+D=0$ sur une surface.

1) Supposons qu'on cherche sur une surface une ligne L pour laquelle il existe dans l'espace une autre courbe L_1 dont tout point et la direction sont liés invariablement au trièdre de Darboux de la ligne sur la surface au point homologue. Dans le cas général l'échelle de représentation $\frac{ds_1}{ds}$ n'est pas constante, c'est pourquoi il est nécessaire que la direction joignant les points homologues M et M_1 soit simultanément perpendiculaire à la direction de la ligne L sur la surface et à la direction de la courbe adjointe L_1. L'équation de la ligne cherchée sur la surface s'obtient dans la forme

$$(\bar{t}_1, \bar{t})\ \frac{1}{T_g}-(\bar{t}_1, \bar{n})\,\nu-(\bar{t}_1, \bar{N})\,\frac{1}{\rho_g}=\frac{(\bar{t}_1, \bar{n})}{d(\bar{d}^\circ, \bar{N})}$$

où $\bar{t}$, t_1 sont les vecteurs unitaires des tangentes aux lignes L, L_1; $\bar{n}$ est le vecteur unitaire de la normale géodésique de la ligne L; $\bar{N}$ est le vecteur unitaire de la normale à la surface; $\bar{d}^\circ$ est le vecteur unitaire de la direction, joignant les points M, M_1; d est la distance entre ces derniers.

Inversement, on peut montrer que toute ligne sur la surface, dont la torsion géodésique, la courbure normale et la courbure géodésique sont liées par une relation linéaire à coefficients constants, admet, en général, une seule courbe adjointe, dont le point et la direction sont liés invariablement au trièdre de Darboux au point homologue de la ligne sur la surface.

2) Outre le cas général, nous avons encore le cas particulier, quand l'échelle de représentation est la valeur constante $\frac{ds_1}{ds}=\frac{(\bar{d}^\circ, \bar{t})}{(\bar{d}^\circ, \bar{t}_1)}$. Dans ce cas la ligne sur la surface se détermine par le système des équations

$$\frac{ds_1}{ds}(\bar{t}_1, \bar{t})=1+d\left(c\nu-b\frac{1}{\rho_g}\right),\ \frac{ds_1}{ds}(\bar{t}_1, \bar{n})=d\left(a\frac{1}{\rho_g}+c\frac{1}{T_g}\right)$$

où $\bar{d}^\circ = a\bar{t} + b\bar{n} + c\bar{N}$. La courbure et la torsion ordinaires d'une telle ligne sont liées par une seule relation.

II. Cas particuliers de la classe des lignes $A\frac{1}{T_g} + B\nu + C\frac{1}{\rho_g} + D = 0$ sur la surface.

1) La direction de la courbe adjointe est parallèle au plan tangent à la surface au point homologue de la ligne L. La ligne L sur la surface aura dans ce cas la forme

$$\frac{\operatorname{ctg}\psi}{T_g} - \nu = \frac{1}{d}, \text{ où } \cos\psi = (\bar{t}_1, \bar{t}).$$

Ce cas a été considéré dans notre Note précédente (1).

2) La direction de la courbe adjointe est perpendiculaire à la normale géodésique de la ligne L. La ligne L sur la surface aura dans ce cas la forme $\frac{\operatorname{ctg}\psi}{T_g} - \frac{1}{\rho_g} = \frac{1}{d}$. En particulier, si $\psi = 90°$, on obtient sur la surface la ligne $\rho_g = d = \text{const}$. Si $\psi = 90°$ et $d \to \infty$, on obtient les lignes géodésiques. Si la condition $d \to \infty$ est seule remplie, la ligne sur la surface aura la forme

$$\frac{1}{T_g} : \frac{1}{\rho_g} = \text{const}.$$

3) La direction de la courbe adjointe est perpendiculaire à la direction de la ligne sur la surface. Dans ce cas on obtient sur la surface des lignes de forme $B\nu + C\frac{1}{\rho_g} + D = 0$. En particulier, quand $d \to \infty$ on obtient les lignes $\theta = \text{const}$, où θ est l'angle entre la normale à la surface et la normale principale de la courbe.

4) La direction de la courbe adjointe est parallèle à celle de la ligne sur la surface; on obtient alors les lignes de courbure de la surface. Les rayons de courbure et de torsion de la courbe adjointe et des lignes de courbure sont respectivement proportionnels (les uns aux autres).

5) La courbe adjointe s'éloigne à l'infini, alors la ligne sur la surface est de forme $A\frac{1}{T_g} + B\nu + C\frac{1}{\rho_g} = 0$. Si la ligne sur la surface admet 2 courbes adjointes à l'infini, elle sera de forme $\frac{1}{T_g} : \nu = p = \text{const}$, $\frac{1}{\rho_g} : \nu = q = \text{const}$. Cette ligne est une hélice $\frac{\rho}{T} = \text{const}$.

III. La courbe adjointe.

1) A toute ligne de la classe $A\frac{1}{T_g} + B\nu + C\frac{1}{\rho_g} + D = 0$ (*) sur la surface correspond dans l'espace sa courbe adjointe, dont le point et la direction sont liés invariablement au trièdre de Darboux de la ligne L au point homologue. La courbe adjointe se détermine de l'équation $A\frac{1}{T_g} + B\nu + C\frac{1}{\rho_g} + D = 0$ par les formules:

$$(\bar{t}_1, \bar{t}) = \frac{A}{\sqrt{A^2 + B^2 + C^2}}, (\bar{t}_1, \bar{n}) = -\frac{B}{\sqrt{A^2 + B^2 + C^2}}, (\bar{t}_1, \bar{N}) = -\frac{C}{\sqrt{A^2 + B^2 + C^2}},$$

$$d = \frac{\sqrt{B^2 + C^2}}{D}, (\bar{d}^\circ, \bar{t}) = 0, (\bar{d}^\circ, \bar{n}) = -\frac{C}{\sqrt{B^2 + C^2}}, (\bar{d}^\circ, \bar{N}) = \frac{B}{\sqrt{B^2 + C^2}}.$$

2) Nous pouvons considérer la courbe adjointe comme située sur une certaine bande de la surface; il est commode de choisir cette dernière de manière que le trièdre de Darboux soit le long de la courbe adjointe

lié invariablement au trièdre de Darboux de la ligne L au point homologue (ce qu'on peut faire de ∞^1 manières); les rôles de la ligne (*) et de la courbe adjointe seront alors semblables.

3) Les torsions géodésiques de la ligne (*) sur la surface et de la courbe adjointe, relative à la bande de la surface, sont liées par les relations:

$$\frac{1}{T_g}\left(\frac{1}{T_g}\right)_1 = \left(\frac{\sin\psi}{d}\right)^2 = \text{const}, \quad \left(\frac{ds_1}{ds}\right)^2 = \frac{1}{T_g} : \left(\frac{1}{T_g}\right)_1.$$

4) Si l'on projette la ligne (*) de la surface sur le plan défini par la tangente $\bar{t}$ et la ligne de liaison $\bar{d}^\circ$, et la courbe adjointe sur le plan $\bar{d}^\circ$, $\bar{t}_1$, les rayons de courbure des projections aux points homologues sont liés par une relation $r_1 = \frac{d(r+d)}{d + r\sin^2\psi}$. On peut donner à cette relation la forme suivante:

$$(MM_1CC_1) = \sec^2\psi = \text{const}$$

où C, C_1 sont les centres de courbure des projections.

5) La torsion géodésique, la courbure normale et la courbure géodésique d'une bande adjointe de la surface s'expriment par les mêmes éléments de la ligne (*) sur la surface de la manière suivante:

$$\left(\frac{1}{T_g}\right)_1 = \left(\frac{\sin\psi}{d}\right)^2 T_g; \quad d\nu_1 = c_1\left[1 - \frac{\cos^2\psi}{1 + d\left(c\nu - b\frac{1}{\rho_g}\right)}\right] -$$

$$- b_1 \sin\psi\, T_g\left(b\nu + c\frac{1}{\rho_g}\right);$$

$$-d\left(\frac{1}{\rho_g}\right)_1 = c_1 \sin\psi\, T_g\left(b\nu + c\frac{1}{\rho_g}\right) + b_1\left[1 + \frac{\cos^2\psi}{1 + d\left(c\nu - b\frac{1}{\rho_g}\right)}\right].$$

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LITTÉRATURE CITÉE

[1] N. Touganoff, C. R. Acad. Sci. URSS, XX, No. 7—8 (1938).

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MATHÉMATIQUES

SUR LA FORTE CONVERGENCE DES PROCÉDÉS D'INTERPOLATION

Par S. LOZINSKI

(Présenté par S. N. Bernstein, de l'Académie, le 18. XI. 1940)

Dans la première partie de cette Note on étudie les polynômes trigonométriques qui interpolent une fonction donnée en des points équidistants et les moyennes de Fejér de ces polynômes; dans la seconde partie sont établis quelques résultats négatifs, analogues au théorème connu de G. Faber.

I. Soit $f(x)$ une fonction réelle, définie sur l'intervalle $0 \leqq x \leqq 2\pi$. Désignons par $U_n(f) = U_n(f,x)$ le polynôme trigonométrique d'ordre $\leqq n$ qui prend les mêmes valeurs que $f(x)$ aux points

$$x_k = x_k^{(n)} = \frac{2k\pi}{2n+1} \qquad (k = 1, 2, \ldots, 2n+1; \quad n = 1,2,\ldots).$$

Il est connu qu'un tel polynôme existe et qu'il est unique. Son expression explicite est la suivante:

$$U_n(f,x) = \sum_{k=1}^{2n+1} f(x_k) \frac{\sin (2n+1)\frac{x-x_k}{2}}{(2n+1)\sin\frac{x-x_k}{2}}.$$

Écrivons le polynôme $U_n(f, x)$ sous la forme

$$U_n(f, x) = a_0^{(n)} + \sum_{\nu=1}^{n} (a_\nu^{(n)} \cos \nu x + b_\nu^{(n)} \sin \nu x)$$

et posons

$$U_{n,i}(f, x) = \begin{cases} a_0^{(n)} + \sum_{\nu=1}^{i} (a_\nu^{(n)} \cos \nu x + b_\nu^{(n)} \sin \nu x) & (i = 1, 2, \ldots n) \\ a_0^{(n)} \qquad \text{pour } i = 0 \end{cases}$$

$$V_n(f) = V_n(f, x) = \frac{1}{n+1} \sum_{i=0}^{n} U_{n,i}(f, x).$$

Les polynômes $V_n(f, x)$ ont été introduits par S. Bernstein (1). Ils sont donnés par la formule:

$$V_n(f, x) = \frac{1}{2n+1} \sum_{k=1}^{2n+1} f(x_k) \frac{1}{n+1} \left[\frac{\sin (n+1)\frac{x-x_k}{2}}{\sin\frac{x-x_k}{2}} \right]^2.$$

Soit $\tilde{A}$ l'ensemble des fonctions $f(x)$ absolument continues sur l'intervalle $[0, 2\pi]$ et satisfaisant à la condition $f(0)=f(2\pi)$. Soit $L^p(p>0)$, comme d'ordinaire, l'ensemble de toutes les fonctions $f(x)$ mesurables sur $[0, 2\pi]$ et telles que $|f(x)|^p$ est sommable. Soit enfin Z l'ensemble de toutes les fonctions $f(x)$ mesurables sur $[0, 2\pi]$ et telles que $|f|\log^+|f|$ est sommable.

Posons

$$U_n'(f, x)=\frac{d}{dx}U_n(f, x); \qquad V_n'(f, x)=\frac{d}{dx}V_n(f, x).$$

Marcinkiewicz (²) a démontré les théorèmes suivants concernant les polynômes $U_n(f)$ et $V_n(f)$:

I. *Si* $f(x)\in\tilde{A}$ *et* $f'(x)\in L^p(p>1)$, *alors*

$$\lim_{n\to\infty}\int_0^{2\pi}|f'-U_n'(f)|^p dx=0.$$

II. *Soit* $f(x)\in\tilde{A}$. *Alors on a*

$$\lim_{n\to\infty}V_n'(f, x)=f'(x)$$

presque partout sur $[0, 2\pi]$.

Ces théorèmes sont analogues aux résultats bien connus de la théorie des séries de Fourier. Cette analogie peut être poursuivie plus loin. On a les théorèmes suivants:

Théorème 1. *Soit* $f(x)\in\tilde{A}$ *et* $f'(x)\in Z$. *Alors on a*

$$\lim_{n\to\infty}\int_0^{2\pi}|f'-U_n'(f)|\,dx=0. \tag{1}$$

Il existe [(³), ou bien théorème 10 de cette Note] une fonction $f(x)\in\tilde{A}$, pour laquelle (1) n'a pas lieu.

Théorème 2. *Soit* $f(x)\in\tilde{A}$. *Alors on a*

$$\lim_{n\to\infty}\int_0^{2\pi}|f'-V_n'(f)|\,dx=0. \tag{2}$$

La démonstration du théorème 2 s'appuie sur le

Théorème 3. *Soit* $f(x)\in\tilde{A}$. *Il existe alors une suite de polynômes trigonométriques* $\{t_n(x)\}_{n=1,2\ldots}$ *d'ordre, respectivement,* $\leqq n$, *telle que*

$$\int_0^{2\pi}|f-t_n|\,dx=o\left(\frac{1}{n}\right) \tag{3}$$

$$\int_0^{2\pi}|f'-t_n'|\,dx=o(1). \tag{4}$$

Soit $M(u)$ une fonction définie pour $u\geqq 0$ et satisfaisant aux conditions suivantes:

1) $M(0)=0$, $M(u)\geqq 0$ et $M(u)$ est une fonction convexe et croissante (au sens strict) pour $0\leqq u<+\infty$.

2) Il existe une constante $K>0$ telle que $M(2u)\leqq KM(u)$ pour $0\leqq u<+\infty$.

3) $\lim_{u\to+0}\frac{M(u)}{u}=0;\qquad \lim_{u\to+\infty}\frac{M(u)}{u}=+\infty.$

Désignons par L^M l'ensemble des fonctions $f(x)$ mesurables sur $[0,2\pi]$ et telles que

$$\int_0^{2\pi} M[|f(x)|]\,dx<+\infty.$$

Alors on a le

Théorème 4. *Soit* $f(x)\in\tilde{A}$ *et* $f'(x)\in L^M$. *Alors il existe une suite de polynômes trigonométriques* $\{t_n(x)\}_{n=1,2\ldots}$ *d'ordre, respectivement,* $\leqq n$, *et telle que*

$$\lim_{n\to\infty}\int_0^{2\pi} M(n|f-t_n|)\,dx=0, \tag{5}$$

$$\lim_{n\to\infty}\int_0^{2\pi} M(|f'-t_n'|)\,dx=0. \tag{6}$$

Remarque. Soit $f(x)$ une fonction satisfaisant aux conditions du théorème 4. Alors chaque suite de polynômes $\{t_n(x)\}_{n=1,2\ldots}$ (d'ordre, respectivement, $\leqq n$) qui satisfait à la condition (5) satisfait aussi à (6).

On a une remarque analogue concernant le théorème 3.

Le théorème 1 résulte du théorème 4. Il résulte encore du théorème 4 que si $f(x)\in\tilde{A}$, $f'(x)\in L^M$, alors

$$\lim_{n\to\infty}\int_0^{2\pi} M(|f'-V_n'(f)|)\,dx=0.$$

A l'aide des théorèmes 3 et 4 on trouve le théorème suivant concernant l'approximation en moyenne d'une fonction $f(x)$ absolument continue par les sommes partielles $S_n(f)=S_n(f,x)$ de la série de Fourier de cette fonction et par les polynômes $U_n(f,x)$.

Théorème 5. *Soit* $f(x)\in\tilde{A}$. *Alors on a*

$$\int_0^{2\pi}|f-S_n(f)|\,dx=o\left(\frac{\log n}{n}\right),\qquad \int_0^{2\pi}|f-U_n(f)|\,dx=o\left(\frac{\log n}{n}\right).$$

Soit $f(x)\in\tilde{A}$ *et* $f'(x)\in Z$. *Alors on a*

$$\int_0^{2\pi}|f-S_n(f)|\,dx=o\left(\frac{1}{n}\right),\qquad \int_0^{2\pi}|f-U_n(f)|\,dx=o\left(\frac{1}{n}\right).$$

Du théorème 2 on tire le

Théorème 6. *Il existe une constante absolue* $C>0$ *telle que pour toute fonction* $f(x)$ *définie sur* $[0,2\pi]$ *on a*

$$\operatorname*{var}_{0\leqq x\leqq 2\pi} V_n(f,x)\leqq C\operatorname*{var}_{0\leqq x\leqq 2\pi} f(x)\qquad (n=1,2\ldots).$$

Du théorème 6 résulte le

Théorème 7. *Soit* $f(x)$ *une fonction continue et de période* 2π. *Pour que cette fonction soit à variation bornée sur* $[0,2\pi]$ *il faut et il suffit qu'il existe une constante* $K>0$ *telle que*

$$\operatorname*{var}_{0\leqq x\leqq 2\pi} V_n(f,x)\leqq K<+\infty\qquad (n=1,2\ldots). \tag{7}$$

En généralisant le théorème 7 on trouve la proposition suivante.

Théorème 8. *Soit $f(x)$ une fonction réelle de la variable réelle x, de période 2π et satisfaisant en chaque point x à la condition*

$$\liminf_{\substack{\xi\to x\\ \xi\neq x}} f(\xi) \leqq f(x) \leqq \limsup_{\substack{\xi\to x\\ \xi\neq x}} f(\xi).$$

Supposons qu'il existe une constante $K>0$ satisfaisant aux conditions (7). Alors deux cas seulement sont possibles: 1) $f(x)$ est à variation bornée sur $[0, 2\pi]$. 2) L'ensemble des points de discontinuité de la fonction $f(x)$ est de la puissance du continu.

Pour terminer cette section, indiquons encore le

Théorème 9. *Soit $f(x)$ une fonction à variation bornée sur $[0, 2\pi]$. Alors pour que la fonction $f(x)$ soit continue et satisfasse à la condition $f(0)=f(2\pi)$, il faut et il suffit que*

$$\int_0^{2\pi} |f - S_n(f)|^2 dx = o\left(\frac{1}{n}\right).$$

II. Soit

$$\begin{array}{l} x_1^{(1)} \\ x_1^{(2)} \quad x_2^{(2)} \quad x_3^{(2)} \\ \cdots\cdots\cdots\cdots \\ x_1^{(n)} \quad x_2^{(n)} \quad \ldots \quad x_{2n+1}^{(n)} \\ \cdots\cdots\cdots\cdots \end{array} \tag{8}$$

un tableau de points d'interpolation, situé sur $[0, 2\pi]$

$$(0 \leqq x_1^{(n)} < x_2^{(n)} < \ldots < x_{2n+1}^{(n)} \leqq 2\pi, \quad x_{2n+1}^{(n)} - x_1^{(n)} < 2\pi; \quad n=1,2\ldots).$$

Désignons par ξ_n le système de points

$$x_1^{(n)}, \quad x_2^{(n)}, \ldots x_{2n+1}^{(n)}.$$

Soit $U_n(\xi_n, f, x)$ le polynôme trigonométrique d'ordre $\leqq n$ qui coïncide avec $f(x)$ aux points du système ξ_n. Alors un théorème connu de Faber affirme, que:

Quel que soit le tableau des points d'interpolation (8) (*satisfaisant aux conditions précédemment nommées*), *il existe une fonction continue $f(x)$ de période 2π telle, que la relation*

$$\lim_{n\to\infty} U_n(\xi_n, f, x) = f(x)$$

n'a pas lieu uniformément sur $[0, 2\pi]$.

On a la proposition analogue suivante:

Théorème 10. *Quel que soit le système des points d'interpolation* (8), *il existe une fonction absolument continue $f(x)$ de période 2π telle que la relation*

$$\lim_{n\to\infty} \int_0^{2\pi} |f' - U_n'(\xi_n, f)| dx = 0$$

n'a pas lieu.

Un théorème analogue au théorème 10 a lieu pour l'interpolation du moyen de polynômes.

Soit (8) un système de points d'interpolation satisfaisant aux conditions nommées. Faisons correspondre à chaque point $x_i^{(n)}$ $(i=1, 2, \ldots, 2n+1, \; n=1, 2\ldots)$ un intervalle fermé $\delta_i^{(n)}$ de telle façon, que $x_i^{(n)} \in \delta_i^{(n)}$, les intervalles $\delta_1^{(n)}, \delta_2^{(n)}, \ldots \delta_{2n+1}^{(n)}$ n'empiètent pas, appartiennent tous à

l'intervalle $[0, 2\pi]$ et couvrent tout l'intervalle $[0, 2\pi]$. Désignons par $\delta_i^{(n)}$ non seulement l'intervalle $\delta_i^{(n)}$, mais aussi sa longueur; un malentendu n'est pas à craindre. Le polynôme interpolateur trigonométrique peut être écrit sous la forme

$$U_n(\xi_n, f, x) = \sum_{i=1}^{2n+1} f(x_i^{(n)})\, l_i^{(n)}(x)$$

où $l_i^{(n)}(x)$ sont les polynômes fondamentaux de l'interpolation trigonométrique, définis par les conditions

$$l_i^{(n)}(x_j^{(n)}) = \begin{cases} 1 & \text{pour } i = j \\ 0 & \text{pour } i \neq j \end{cases} \quad \begin{pmatrix} i, j = 1,2,\ldots,2n+1 \\ n = 1, 2\ldots \end{pmatrix}$$

Faisons correspondre au polynôme $U_n(\xi_n, f, x)$ le polynôme

$$Q_n(\xi_n, f, x) = \sum_{i=1}^{2n+1} \frac{1}{\delta_i^{(n)}} \int_{\delta_i^{(n)}} f(t)\, dt\; l_i^{(n)}(x).$$

On peut former les polynômes $Q_n(\xi_n, f, x)$ pour toute fonction sommable $f(x)$ [voir(4)]. On a le

Théorème 11. *Quel que soit le système de points d'interpolation* (8) *et quel que soit le choix des intervalles* $\delta_i^{(n)}$, *il existe une fonction* $f(x)$ *sommable sur* $[0, 2\pi]$ *telle que la relation*

$$\lim_{n\to\infty} \int_0^{2\pi} |f - Q_n(\xi_n, f)|\, dx = 0$$

n'a pas lieu.

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MATHÉMATIQUES

SUR LES FORMULES SOMMATOIRES ET LES IDENTITÉS ANALYTIQUES, LIÉES À UNE CLASSE DE FONCTIONS ARITHMÉTIQUES

Par E. A. ANFERTIEVA

(Présenté par S. N. Bernstein, de l'Académie, le 13. XI. 1940)

Dans une série de Notes, publiées dans les Comptes Rendus de l'Académie des Sciences de l'URSS, N. S. Košljakov [1] a étudié les formules sommatoires et quelques identités analytiques, liées aux séries de Dirichlet.

Dans le présent article l'auteur se propose à montrer quelques résultats analogues obtenus pour les fonctions arithmétiques

$$a_\nu(n) = \frac{\sum_{d\delta=n} d^{2\nu}}{n^\nu}$$

définies par la série de Dirichlet suivante:

$$\varphi_\nu(s) = \zeta(s+\nu)\,\zeta(s-\nu) = \sum_{n=1}^{\infty} \frac{a_\nu(n)}{n^s},$$

$$s = \tau + it, \quad R(s) > 1+\nu, \quad -\frac{1}{2} < \nu < \frac{1}{2}$$

où $\zeta(s)$ est la fonction connue de Riemann.

Il faut remarquer, que pour la fonction $\varphi_\nu(s)$ a lieu l'équation fonctionnelle

$$\Gamma\left(\frac{s+\nu}{2}\right)\Gamma\left(\frac{s-\nu}{2}\right)\varphi_\nu(s) = \pi^{2s-1}\,\Gamma\left(\frac{1-s-\nu}{2}\right)\Gamma\left(\frac{1-s+\nu}{2}\right)\varphi_\nu(1-s).$$

L'auteur a établi les égalités suivantes:

1. En posant $\psi(t) = \xi\left(\frac{1}{2}+it\right)$ où

$$\xi(s) = \frac{(s-\nu)(s+\nu)[1-(s-\nu)][1-(s+\nu)]}{\pi^s}\,\Gamma\left(\frac{s-\nu}{2}\right)\Gamma\left(\frac{s+\nu}{2}\right)\varphi_\nu(s)$$

on aura la représentation intégrale de la fonction

$$\psi(t) = \frac{\pi}{\sin \pi\nu}\left[-\frac{\zeta(2\nu)\,\pi^{-\nu}(1-2\nu)}{\Gamma(1-\nu)}\left\{\left(\frac{1}{2}+\nu\right)^2+t^2\right\}+\right.$$

$$+\frac{\zeta(-2\nu)\,\pi^{\nu}(1+2\nu)}{\Gamma(1+\nu)}\left\{\left(\frac{1}{2}-\nu\right)^2+t^2\right\}+$$

$$\left.+4\left\{\left(\frac{1}{2}-\nu\right)^2+t^2\right\}\left\{\left(\frac{1}{2}-\nu\right)^2+t^2\right\}\int_1^\infty x^{-\frac{1}{2}}\psi(x)\cos(t\log x)\,dx\right. \quad \text{(I)}$$

2. Si l'on désigne $\varepsilon = e^{\frac{i\pi}{4}}$, $\bar{\varepsilon} = e^{-\frac{i\pi}{4}}$, nous aurons le développement

$$\pi^{-s}\Gamma\left(\frac{s-\nu}{2}\right)\Gamma\left(\frac{s+\nu}{2}\right)\zeta(s-\nu)\zeta(s+\nu) = U(\varepsilon, s) + U(\bar{\varepsilon}, 1-s) \qquad (II)$$

où

$$U(\varepsilon, s) = \frac{\Gamma(\nu)\zeta(2\nu)}{\pi^{\nu}(\nu - s)}\varepsilon^{s-\nu} - \frac{\Gamma(-\nu)\zeta(-2\nu)}{\pi^{-\nu}(\nu + s)}\varepsilon^{s+\nu} +$$

$$+ 4\varepsilon^{s}\sum_{n=1}^{\infty} a_{\nu}(n)\int_{1}^{\infty} K_{\nu}(2\pi\varepsilon n x)\, x^{s-1}\, dx$$

où $K_{\nu}(x)$ est une fonction cylindrique. Nous pouvons donc écrire ce développement sous la forme suivante:

$$\pi^{-s}\Gamma\left(\frac{s-\nu}{2}\right)\Gamma\left(\frac{s+\nu}{2}\right)\zeta(s-\nu)\zeta(s+\nu) =$$

$$= \Gamma\left(\frac{s-\nu}{2}\right)\Gamma\left(\frac{s+\nu}{2}\right)\pi^{-s}\int_{0\nwarrow 1}\omega(\bar{\varepsilon}, x)\, x^{-s}\, dx +$$

$$+ \Gamma\left(\frac{s-\nu}{2}\right)\Gamma\left(\frac{s+\nu}{2}\right)\pi^{-s}\int_{0\swarrow 1}\omega(\varepsilon, x)\, x^{s-1}\, dx \qquad (II_1)$$

où

$$\omega(\bar{\varepsilon}, x) = \frac{1}{2\pi i}\left\{-\frac{\Gamma(\nu)\zeta(2\nu)}{\pi^{\nu}}\bar{\varepsilon}^{1+3\nu}\pi J_{1-\nu}(2\pi\bar{\varepsilon}x) - \right.$$

$$\left. - \frac{\Gamma(-\nu)\zeta(-2\nu)}{\pi^{-\nu}}\bar{\varepsilon}^{1-3\nu}\pi J_{1+\nu}(2\pi\bar{\varepsilon}x) + 2x\varepsilon^{2\nu}\sum_{n=1}^{\infty}\frac{X(2\pi\bar{\varepsilon}x, 2\pi\varepsilon n)}{x^2 - n^2}\right\},$$

$$X(2\pi\bar{\varepsilon}x, 2\pi\varepsilon n) = K_{\nu}(2\pi\varepsilon n)\, J_{\nu+1}(2\pi\bar{\varepsilon}x)\, 2\pi\varepsilon x - J_{\nu}(2\pi\bar{\varepsilon}x)\, K_{\nu+1}(2\pi\varepsilon n)\, 2\pi\varepsilon n;$$

$J_{\nu}(x)$ est la fonction de Bessel.

Les intégrales de la formule (II_1) sont prises le long des droites parallèles aux bissectrices des angles formés par les axes de coordonnées et coupant l'axe réel entre 0 et 1. La propriété indiquée de la fonction $\zeta(s-\nu)\zeta(s+\nu)$ est analogue à la formule connue de Riemann-Siegel [2].

3. En posant $M_{2\nu}(x) = \frac{2}{\pi}K_{2\nu}(x) - Y_{2\nu}(x)$, on obtiendra la relation

$$\zeta(s+\nu)\zeta(s-\nu) = \sum_{n=1}^{\infty}\frac{a_{\nu}(n)\, n^{-s}}{1+\left(\frac{n}{x}\right)^{s}} - \frac{\pi x^{1+s-\nu}}{2\cos\frac{s-\nu}{2}\pi}\zeta(1-2\nu) -$$

$$- \frac{\pi x^{1-s-\nu}}{2\cos\frac{s+\nu}{2}\pi}\zeta(1+2\nu) + 2\pi x^{1-s}\int_{0}^{\infty}\left[\sum_{n=1}^{\infty}\frac{a_{\nu}(n)\, n^{s-1}}{1+\left(\frac{n}{y}\right)^{2}} - \frac{\pi y^{s+\nu}}{2\sin\frac{s+\nu}{2}\pi}\zeta(1-2\nu) - \right.$$

$$\left. - \frac{\pi y^{s-\nu}}{2\sin\frac{s-\nu}{2}\pi}\zeta(1+2\nu)\right]\left[\frac{\cos\nu\pi M_{2\nu}(4\pi\sqrt{xy}) - \sin\nu\pi J_{2\nu}(4\pi\sqrt{xy})}{y^{s}}\right] dy,$$

$$x > 0, \qquad (III)$$

laquelle est analogue à la formule de N. S. Košljakov pour la fonction de Riemann.

4. Désignons par $F(s)$ la fonction de la variable complexe, soumise à certaines conditions; on aura la formule sommatoire:

$$\sum_{n=1}^{\infty} a_{\nu}(n)\, f(n) = \varphi_{\nu}(\nu)\, R_{s=\nu}[F(s)] + \varphi_{\nu}(-\nu)\, R_{s=-\nu}[F(s)] -$$

$$-2\int_0^\infty f(x)\left\{\frac{\varphi_\nu(\nu)\,\Gamma(2\nu)}{\pi}(2\pi)^{1-2\nu}x^{-\nu}+\frac{\varphi_\nu(-\nu)\,\Gamma(-2\nu)}{\pi}(2\pi)^{1+2\nu}x^{\nu}\right\}\cos\pi\nu\,dx-$$

$$-2\int_0^\infty \frac{e^{\frac{\nu\pi i}{2}}f(ix)-e^{-\frac{\nu\pi i}{2}}f(-ix)}{2i}\sigma_\nu(x)\,dx \qquad \text{(IV)}$$

où $$\sigma_\nu(x)=2\sum_{n=1}^{\infty}a_\nu(n)\{e^{\frac{\nu\pi i}{2}}K_{2\nu}(4\pi\sqrt{nx})+e^{-\frac{\nu\pi i}{2}}K_{2\nu}(4\pi\sqrt{nx})\}-$$

$$=-\frac{\varphi_\nu(\nu)}{\pi}x^{\nu-1}+\frac{\varphi_\nu(\nu)\,\Gamma(2\nu)}{\pi}(2\pi)^{1-2\nu}x^{-\nu}+$$

$$+\frac{\varphi_\nu(-\nu)\,\Gamma(-2\nu)\cos\pi\nu}{\pi}(2\pi)^{1+2\nu}x^{\nu}+\frac{x^{\nu+1}}{\pi}\sum_{n=1}^{\infty}\frac{a_\nu(n)}{n^\nu}\cdot\frac{1}{x^2+n^2};$$

$R_{\nu,\pm\nu}[F(s)]$ désignent les résidus de la fonction $F(s)$ correspondant aux pôles $s=\pm\nu$.

5. Aussi on peut démontrer les formules analogues à celles de Poisson et de Voronoï [3]:

$$\sum_{n=1}^{\infty}a_\nu(n)f(n)=R_{1+\nu}+R_{1-\nu}+R_\nu+R_{-\nu}+$$

$$+2\pi\sum_{n=1}^{\infty}a_\nu(n)\int_0^\infty f(x)\{\cos\nu\pi M_{2\nu}(4\pi\sqrt{nx})-\sin\nu\pi J_{2\nu}(4\pi\sqrt{nx})\}\,dx; \qquad \text{(V)}$$

$$\sum_{n=1}^{\nu<\beta}{}'a_\nu(n)f(n)=\int_\alpha^\beta f(\vartheta)\left\{\frac{2\Gamma(2\nu)\,\zeta(2\nu)}{(2\pi)^{-2\nu}}\vartheta^{-\nu}+\frac{2\Gamma(-2\nu)\,\zeta(-2\nu)}{(2\pi)^{2\nu}}\vartheta^{\nu}\right\}\cos\pi\nu\,d\vartheta+$$

$$+2\pi\sum_{n=1}^{\infty}a_\nu(n)\int_\alpha^\beta f(\vartheta)\{\cos\nu\pi M_{2\nu}(4\pi\sqrt{n\vartheta})-\sin\nu\pi J_{2\nu}(4\pi\sqrt{n\vartheta})\}\,d\vartheta \qquad \text{(VI)}$$

où $R\mu$ est le résidu de la fonction $\varphi_\nu(s)F(s)$ au point $s=\mu$ ($\mu=1-\nu$, $1+\nu$, ν, $-\nu$); et l'accent affectant la somme signifie la nécessité de prendre le dernier terme avec le coefficient 1/2, dans le cas où β est entier.

La démonstration détaillée de toutes ces formules sera publiée ailleurs.

Pour obtenir la formule (IV) l'auteur emploie la méthode indiquée par Košljakov dans son article, qui va paraître dans le Bulletin de l'Académie des Sciences de l'URSS. Les formules (V) et (VI) sont obtenues par la méthode de Ferrar [4] à l'aide de la théorie de l'intégrale de Stieltjes.

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MATHÉMATIQUES

SUR QUELQUES PROPRIÉTÉS GÉOMÉTRIQUES DE LA SPHÈRE DANS LES ESPACES LINÉAIRES SEMI-ORDONNÉS DE BANACH

Par V. ŠMULIAN

(Présenté par A. N. Kolmogoroff, de l'Académie, le 30. XI. 1940)

Soit E un certain espace linéaire normé, et $K \subset E$ un certain ensemble jouissant des propriétés suivantes *:

1°. Si $x \in K$, on a $\lambda x \in K$ pour $\lambda \geqslant 0$.

2°. Si $x, y \in K$, on a $x+y \in K$.

L'ensemble de toutes les $f \in \bar{E}$, non négatives pour $x \in K$, forme un certain ensemble $\bar{K}$ jouissant des propriétés 1°, 2°. On définit d'une manière analogue $\bar{\bar{K}} \subset \bar{\bar{E}} = (\bar{\bar{E}})$.

Désignons par Q l'intersection de la sphère unitaire $\|x\| \leqslant 1$ avec l'ensemble K.

§ 1. Dans ce paragraphe nous allons supposer que l'ensemble K jouit des propriétés suivantes:

3°. Chaque élément $x \in E$ admet un développement $x = x_1 - x_2$ (x_1, $x_2 \in K$).

4°. L'ensemble K est fermé.

On peut montrer que dans ce cas pour chaque $x \in E$ il existe un développement $x = x_1' - x_2'$ ($x_1', x_2' \in K$) tel que $\|x_1'\| + \|x_2'\| \leqslant C\|x\|$, où C ne dépend das de x. Il en résulte que la sphère unitaire $\|x\| \leqslant 1$ et l'ensemble Q ne peuvent être faiblement compacts que simultanément.

Pour chaque $F \in \bar{\bar{E}}$ on a

$$|F(f)| \leqslant \|F\| \cdot \|f\| \leqslant \|F\| \cdot C \sup_{x \in Q} |f(x)| \qquad (f \in \bar{E}) \tag{1}$$

parce que $\frac{1}{c}\|f\| \leqslant \sup_{x \in Q} |f(x)| \leqslant \|f\|$ $(f \in \bar{E})$.

En vertu d'un théorème de Hildebrandt (2) il suit de la formule (1) que

$$F(f) = \int_Q f(x)\, d\Phi(e) \qquad (f \in \bar{E}) \tag{2}$$

où $\Phi(e)$ est une fonction d'ensembles $e \subset Q$ additive et bornée. La fonction $\Phi(e)$ peut être présentée sous la forme d'une différence

$$\Phi(e) = \Phi_1(e) - \Phi_2(e)$$

* Ces ensembles ont été étudiés récemment par M. Krein (1).

de deux fonctions additives non négatives. Donc

$$F(f)=\int_Q f(x)\,d\Phi_1(e)-\int_Q f(x)\,d\Phi_2(e). \qquad (f\in\bar{E}) \qquad (3)$$

La formule (3) montre que l'étude des fonctionnelles $F(f)$ se réduit à l'étude des fonctionnelles $G(f)=\int_Q f(x)\,d\Phi(e)$, où $\Phi(e)\geqslant 0$ est une fonction additive et $\Phi(Q)=1$.* Posons maintenant

$$dg=\sup_{\substack{f(x)\leqslant 1\\ x\in Q}} G(f);$$

alors pour la fonctionnelle $H(f)=\frac{1}{dg}G(f)$ on a

$$H(f)\leqslant \sup_{x\in Q} f(x) \quad \text{et} \quad \sup_{\substack{f(x)\leqslant 1\\ x\in Q}} H(f)=1. \qquad (4)$$

Il suit de (3) et (4) que chaque $F\in\bar{\bar{E}}$ peut être présentée dans la forme

$$F=c_1F_1-c_2F_2,$$

où

$$F_1, F_2\in k,\ c_1\geqslant 0,\ c_2\geqslant 0,\ F_i(f)\leqslant \sup_{x\in Q} f(x),\ \sup_{\substack{f(x)\leqslant 1\\ x\in K}} F_i(f)=1 \quad (i=1,2).$$

Ainsi l'étude des fonctionnelles $F\in\bar{\bar{E}}$ se réduit à l'étude des fonctionnelles $H(f)$ vérifiant la condition (4).

Lemme 1. *Si la fonctionnelle linéaire $F(f)$ vérifie la condition*

$$F(f)\leqslant \sup_{x\in Q} f(x)$$

alors pour tous les $f_1, \ldots, f_n\in\bar{E}$ et $\varepsilon>0$ il existe un élément $x\in Q$ tel que

$$|F(f_i)-f_i(x)|<\varepsilon. \qquad (1\leqslant i\leqslant n)$$

La démonstration coïncide en essentiel avec celle d'un théorème analytique dû à Helly (3).

En tenant compte de ce lemme nous pouvons établir la proposition suivante.

Théorème 1 (4,5). *Pour que l'espace E soit régulier, il faut et il suffit que pour chaque suite transfinie $\{x_\xi\}\subset Q$ il existe un élément $x_0\in Q$ tel que*

$$\underline{\lim}\, f(x_\xi)\leqslant f(x_0)\leqslant \overline{\lim}\, f(x_\xi). \qquad (f\in\bar{E})$$

Pour que l'espace E soit régulier, il faut et il suffit que chaque suite transfinie décroissante

$$Q\supset Q_1\supset Q_2\supset\ldots\supset Q_\xi\supset\ldots$$

d'ensembles fermés et convexes Q_ξ appartenant à l'ensemble Q possède une intersection non vide.

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* Il suit du résultat obtenu que l'ensemble $\bar{\bar{K}}$ jouit des propriétés 1°—4°. Cette proposition n'a pas encore été obtenue par l'auteur quand J. Grosberg a démontré ce qui suit: Si l'ensemble K possède les propriétés 1°, 2°, 4°, pour qu'il possède la propriété 3° il faut et il suffit que $\bar{\bar{K}}$ jouisse de la propriété 3°. Alors

$$\sup_{x\in E}\frac{\inf_{y-z=x;\ y,\ z\in K}[\|y\|+\|z\|]}{\|x\|}=\sup_{F\in E}\frac{\inf_{H-G=F;H,\ G\in\bar{\bar{K}}}[\|H\|+\|G\|]}{\|F\|}.$$

Lemme 2(6). *Si la fonctionnelle $H(f)$ vérifie les relations* (4), *on a*

$$H(f) = \lim_{\alpha} f(x_\alpha) \qquad (f \in \bar{E})$$

où $\{x_\alpha\} \subset Q$, α *parcourt un ensemble semi-ordonné* $\lim_{\alpha', \alpha''} \left\| \frac{x_{\alpha'} + x_{\alpha''}}{2} \right\| = 1$. *D'ailleurs pour chaque suite dénombrable* $\{f_n\} \subset \bar{E}$ *on a*

$$H(f_n) = \lim_{p} f_n(x_p) \qquad (1 \leqslant n \leqslant +\infty)$$

où $\{x_p\} \subset Q$, $\lim_{m,n} \left\| \frac{x_m + x_n}{2} \right\| = 1$.

Une suite semi-ordonnée $\{x_\alpha\}$ est nommée *frontière*, si $\lim_{\alpha', \alpha''} \left\| \frac{x_{\alpha'} + x_{\alpha''}}{2} \right\| = 1$. $\|x_\alpha\| = 1$, pour tous les α.

Une suite ordonnée $\{x_\alpha\}$ est nommée *faiblement convergente*, s'il existe $\lim_{\alpha} f(x_\alpha)$ pour tous les $f \in \bar{E}$.

On peut maintenant démontrer les théorèmes suivants.

Théorème 2(6). *Pour que la sphère* $\|x\| \leqslant 1$ *soit faiblement compacte, il faut et il suffit que chaque suite dénombrable frontière* $\{x_n\} \subset Q$ *soit un ensemble faiblement compact.*

Théorème 3(6). *Pour que l'espace soit régulier, il faut et il suffit que chaque suite semi-ordonnée frontière* $\{x_\alpha\} \subset Q$, *faiblement convergente, converge faiblement vers un certain élément* x_0.

Si la sphère $\|x\| \leqslant 1$ est faiblement compacte, les derniers résultats peuvent être précisés.

Lemme 3. *Si la sphère* $\|x\| \leqslant 1$ *est faiblement compacte, alors pour chaque fonctionnelle linéaire* $H(f)$ *vérifiant les relations* (4), *on a*

$$H(f) = \int_{Q'} f(x)\, d\Phi(e) \tag{5}$$

où Q' *est un ensemble convexe appartenant à* S (*on désigne par* S *l'intersection des ensembles* K *et* $\mathcal{E}(\|x\| = 1)$; $\Phi(e)$ *est une fonction additive non négative,* $\Phi(Q') = 1$.

Démonstration. Il existe une suite $\{f_n\} \subset \bar{E}$ telle que

$$\begin{cases} H(f_n) \to 1 \\ f_n(x) \leqslant 1. \end{cases} \qquad (x \in Q; \quad 1 \leqslant n < +\infty)$$

En vertu de la faible compacité de la sphère $\|x\| \leqslant 1$ il suit du lemme 1, que pour un certain point $\bar{x} \in Q$ on a

$$f_n(\bar{x}) = H(f_n). \qquad (1 \leqslant n < +\infty)$$

Désignons par Q' l'ensemble de tous les points $x \in Q$ pour lesquels $f_n(x) \to 1$.

L'ensemble Q' contient le point $\bar{x}$, appartient à S et il est convexe. Comme pour chaque système fini $g_1, \ldots, g_m \in \bar{E}$ on peut trouver un point $x \in Q'$ tel que

$$H(g_i) = g_i(x) \qquad (1 \leqslant i \leqslant m)$$

on a

$$H(f) = \lim_{\alpha} f(x_\alpha) \qquad (f \in \bar{E})$$

où $\{x_\alpha\} \subset Q'$ est une certaine suite semi-ordonnée. Il est facile d'obtenir de cette dernière formule la formule (5).

On peut maintenant démontrer la proposition suivante.

Théorème 4(7). *Si la sphère unitaire de l'espace E est faiblement compacte, pour que E soit régulier, il faut et il suffit que pour chaque suite $\{x_\alpha\}$ semi-ordonnée et faiblement convergente, appartenant à un ensemble convexe quelconque $Q' \subset S$, il existe un élément $x_0 \in E$ tel que*

$$\lim_\alpha f(x_\alpha) = f(x_0). \qquad (f \in \overline{E})$$

Si la sphère unitaire de l'espace E est faiblement compacte, pour que E soit régulier, il faut et il suffit que pour chaque suite transfinie $\{x_\xi\}$, appartenant à un ensemble convexe quelconque $Q' \subset S$, il existe un élément $x_0 \in E$ tel que

$$\underline{\lim}\, f(x_\xi) \leqslant f(x_0) \leqslant \overline{\lim}\, f(x_\xi). \qquad (f \in \overline{E})$$

Corollaire. Si la sphère $\|x\| \leqslant 1$ est faiblement compacte, pour que E soit régulier, il suffit que chaque ensemble convexe appartenant à S soit formé d'un seul point.

On obtient facilement du théorème 2 et du dernier corollaire le

Théorème 5(8). *Pour que l'espace E soit régulier, il suffit que chaque suite dénombrable frontière $\{x_n\} \subset Q$ converge faiblement vers un certain élément $x_0 \in E$.*

En tenant compte du lemme 1, on peut démontrer la proposition suivante.

Théorème 6. *Si l'espace $\overline{E}$ est séparable, pour que E soit régulier, il faut et il suffit que l'ensemble Q soit faiblement complet*.*

En terminant le présent paragraphe, indiquons encore un critère pour la compacité faible.

Théorème 7. *L'ensemble Q est faiblement compact dans le cas et dans ce cas seulement, où la condition suivante est remplie: le système d'équations*

$$f_i(x) = c_i \qquad (1 \leqslant i < +\infty)$$

a une solution $x \in Q$ dans le cas et dans ce cas seulement, où l'on a

$$\sum_1^n \lambda_i c_i \leqslant \sup_{x \in Q} \sum_1^n \lambda_i f_i(x)$$

quels que soient les nombres $n, \lambda_1, \ldots, \lambda_n$.

§ 2. Dans ce paragraphe nous allons supposer que l'ensemble K possède, outre les propriétés 1°, 2° encore les propriétés suivantes:

3°. Chaque élément $x \in E$ admet un développement

$$x = x_1 - x_2 \qquad (x_1, x_2 \in K)$$

où $\|x_1\| \leqslant \|x\|$.

4°. Il existe un $\delta > 0$ tel que $\|e_1 + e_2\| > \delta$ pour

$$e_i \in K, \ \|e_i\| = 1. \qquad (i = 1, 2)$$

---

* L'ensemble $\mathfrak{M} \subset E$ est nommé *faiblement complet*, si chaque suite faiblement convergente $\{x_n\} \subset \mathfrak{M}$ converge faiblement vers un certain élément $x_0 \in E$.

Nous dirons que $\|x\|$ *est uniformément fortement dérivable* $\mathfrak{M}$, si pour chaque $\varepsilon > 0$ il existe un $\delta_\varepsilon > 0$ tel que

$$\left| \frac{\|x_0 + h'x\| - \|x_0\|}{h'} - \frac{\|x_0 + h''x\| - \|x_0\|}{h''} \right| < \varepsilon$$

pour tous les $x_0 \in \mathfrak{M}$, $\|x\| \leqslant 1$, $0 < |h'| < \delta_\varepsilon$, $0 < |h''| < \delta_\varepsilon$.

L'auteur a trouvé récemment un critère pour la dérivabilité uniforme forte $\mathfrak{M}$ pour le cas où $\mathfrak{M}$ coïncide avec la sphère $\|x\| \leqslant 1$ [8]. Un critère analogue peut être formulé pour un ensemble $\mathfrak{M}$ arbitraire borné.

Théorème 7 [8]. *Pour que l'espace E soit régulier, il suffit que* $\|x\|$ *soit uniformément fortement dérivable S.*

Démonstration. Il suit des propriétés de l'ensemble K que l'ensemble $\overline{K}$ possède les propriétés $1^0 - 4^0$ [1]. Si l'on a $f \in \overline{K}$

$$\|f\| = \sup_{x \in Q} f(x). \tag{6}$$

Comme $\|x\|$ est uniformément fortement dérivable S, il existe pour chaque $\varepsilon > 0$ un $\delta_\varepsilon > 0$ tel que les relations

$$f(x) \geqslant 1 - \delta_\varepsilon, \; g(x) \geqslant 1 - \delta_\varepsilon, \; x \in S, \; \|f\| = \|g\| = 1$$

entraînent

$$\|f - g\| \leqslant \varepsilon. \tag{7}$$

Si maintenant $f \in \overline{K}$, $g \in \overline{K}$, $\|f\| = \|g\| = 1$ et $\left\| \frac{f+g}{2} \right\| > 1 - \frac{\delta_\varepsilon}{2}$, on a en vertu de (5) un élément $x \in Q$ tel que

$$\frac{f(x) + g(x)}{2} \geqslant 1 - \frac{\delta_\varepsilon}{2}.$$

Alors

$$f(x) \geqslant 1 - \delta_\varepsilon, \quad g(x) \geqslant 1 - \delta_\varepsilon.$$

Donc (7) est remplie. Maintenant il résulte du théorème 5 que l'espace $\overline{E}$ est régulier. Il reste à remarquer que E et $\overline{E}$ ne peuvent être réguliers que simultanément [9].

D'une manière analogue on peut démontrer la proposition suivante.

Théorème 8. *Si la sphère* $\|x\| \leqslant 1$ *est faiblement compacte, pour que l'espace E soit régulier, il suffit que* $\|x\|$ *soit faiblement dérivable en tous les points* $x \in S$.

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PHYSICS

ON THE BROADENING OF RAYLEIGH SCATTERING LINES DEPENDING UPON PRESSURE

By V. L. GINSBURG

(Communicated by S. I. Wawilow, Member of the Academy, 21. XI. 1940)

The origin of wings, accompanying Rayleigh lines in liquids and compressed gases, presents one of the most important problems in the field of light scattering. Investigations of the influence of pressure on wings in the gas provide, naturally, a method of approaching this problem. At low pressure the scattering spectrum near an unshifted line consists of the Rayleigh line and a rotational wing. This case is discussed in full detail in Placzek and Teller's(1) theory, which considers the scattering by an isolated atom. With an increase in pressure the wings experience marked changes studied by Weiler(2) up to the pressure of 60 atmospheres. Under such a pressure the wings in a gas resemble «liquid» wings to a greater extent than ordinary rotational wings. The interpretation of these facts should be preceded by a study of the influence of pressure on the Rayleigh line, since the intensity of the latter is many thousand times greater than that of the rotational wings and therefore even its feeble broadening is conspicuous.

The existing attempts to evaluate the influence of pressure on the Rayleigh line in gas (excluding that of Leontowitch, see below) are erroneous. Thus, Cabannes and Rocard (3) believe the mechanism of broadening of Rayleigh lines to be analogous to that of the broadening of emission lines during non-quenching collisions. In this case the broadening is the result of the influence, exerted upon the atom by alternating fields of particles colliding with the atom. Rousset (4) and later on Sirkar (5) accepted the usual expression for the width of the Rayleigh line in the case of collision broadening $\left(\delta = \frac{1}{\tau_0}\right.$, where τ_0 is the time of the free path$\left.\right)$(6). It can be easily seen, however, that within the limits of applicability of this formula to emission lines, there will be observed no broadening of the Rayleigh lines caused by this phenomenon. Indeed, the formula $\delta = \frac{1}{\tau_0}$ is valid only if the time of impact $\Delta\tau \ll \tau_0$. At pressures considerably exceeding the atmospheric pressure, when this relation does not hold true, the mechanism of collision broadening is ungrounded* and in the intermediate region it needs to be amended to account for asymmetry and shifting of lines. The collision broadening of emission lines occurs as the result of phase change of vibrations of the emitting oscillator at the moment of impact (6). As the phase changes irregularly, the emission of the atom during different free paths is incoherent and the lines broaden. In the case of scattering the situation is different: no change in phase is experienced by vibrations of the oscillator on impact, as they

* Therefore an a priori application of this formula $\delta = \frac{1}{\tau_0}$ to liquids, as this is done by Rousset (4) and Sirkar (5), is inadmissible, even if the formula is true for gases.

are forced by the incident wave of the frequency ω_0; an impact leads to the change of phase and amplitude of proper vibrations of the oscillator, the frequency of which differs essentially from ω_0. Thus, only the scattered emission at the moment of impact will have an amplitude differing from ω_0. However, because of the condition $\frac{\Delta\tau}{\tau_0} \ll 1$, emission may be neglected.

In contrast to the opinion of the authors mentioned above, Kapler (7) assumes, on grounds which are not obvious to the author of the present paper, that the Rayleigh line experiences altogether no broadening when the pressure is increased. This conclusion is also erroneous. This follows from the fact that on passing from a low pressure gas to a compressed gas and liquid, a thin triplet structure of the Rayleigh line should be revealed.

In order to find out the variation of the form of the Rayleigh line with pressure, let us consider the electric field of the scattered light E at some distance from the scattering volume. The dependence of the exciting plane wave upon the time is supposed to be: $E_0 = 0$ at $t < 0$ and $t > T$ and $E_0 = e^{i\omega_0 t}$ for $0 \leqslant t \leqslant T$. Then in the interval $0 \leqslant t \leqslant T$

$$E = C \sum_{k=1}^{N} e^{i(\omega_0 t + \vec{q}\vec{r}_k(t))} \tag{1}$$

where $\vec{q} = \vec{k}_1 - \vec{k}_0 = \frac{4\pi}{\lambda} \sin\frac{\theta}{2} \frac{\vec{q}}{|\vec{q}|}$, where θ is the angle of scattering, $\vec{k}_1$ and $\vec{k}_0$ are the wave vectors of the scatterd and incident light, λ is the wave length, $\vec{r}_k(t)$ is the radius vector of the k-th atom taken with respect to some point of the scattering volume, and C is a constant. All other constants, presenting no interest, will also be denoted by C in the subsequent exposition. Expanding E in Fourier's integral $E = \int_{-\infty}^{+\infty} E(\omega) e^{i\omega t} d\omega$, we find

$$E(\Omega) = C \sum_{k=1}^{N} \int_0^T e^{i(\vec{q}\vec{r}_k(t) - \Omega t)} dt = C \sum_{k=1}^{N} U_k \tag{2}$$

where $\Omega = \omega_0 - \omega$. The intensity of the scattered light $Y = \frac{c}{T} \int_0^T |E|^2 dt = \int_{-\infty}^{+\infty} Y(\Omega)\, d\Omega$. At a greater T, $Y(\Omega) = \frac{1}{T} |E(\Omega)|^2$ and therefore the quantity $Y(\Omega)$ which is of importance to us is

$$Y(\Omega) = \frac{C}{T} \sum_{k,\, e} U_k U_e^* \tag{3}$$

It was assumed already in (1) that the time dependence of the vibrations of the scattering oscillator, considered to be isotropic, is determined by the exciting wave and by the motion of the atom as a whole. This is indeed the case, if the scattering on impact is not taken into account and molecular fields $\left(\text{of the type } V_{12} \sim \frac{1}{r_{12}} \text{ or } V_{12} \sim \frac{1}{r_{12}^2}\right)$ acting at long range are assumed to be absent. If it were be possible to neglect the collisions of atoms (i. e. $\tau_0 \gg T$), then in the entire interval $0 \leqslant t \leqslant T$

$$\vec{q}\vec{r}_k(t) = \frac{4\pi}{\lambda} \sin\frac{\theta}{2} \cdot \xi_k t + \vec{q}\vec{r}(t=0) = \Delta_k t + \varphi_k \tag{4}$$

where ξ_k is the velocity projection of the k-th atom on $\vec{q}$. It is obvious that at greater $T\left(T \gg \frac{2\pi}{\sqrt{\overline{\Delta_k^2}}}\right)$, $U_k U_k^*$ [see equation (3)] tend to $C\delta(\Delta_k - \Omega)T$, where δ is the delta function; terms $U_k U_e^*$ $(k \neq e)$ tend to zero. The summation in (1)—(3) is extended over all the atoms of the gas; the number of atoms for which ξ lies in the interval $\xi, \xi + d\xi$ equals $f(\xi)\,d\xi = Ce^{-\mu\xi^2/2kT}d\xi$. Therefore, using (4) we find

$$Y(\Omega) = Ce^{-\Omega^2/b^2} \tag{5}$$

where

$$b^2 = \frac{8kT\omega_0^2 \sin^2\frac{\theta}{2}}{\mu C^2}. \tag{6}$$

Case (5) is possible only in a rarefied gas (if $T = 10^{-8}$ sec., then the pressure should be $p < 0.01$ mm Hg). If, however, $T > \tau_0$, then collisions of atoms are to be taken into account. No phase change is observed, and, as it has been already pointed out, no Lorentz broadening takes place in the case of scattering. However, on colliding the atom changes its velocity, this leading to a certain broadening of scattering lines, dependent upon pressure; this broadening is genetically related to Doppler broadening *.

At the moment of impact the phase derivative $\vec{q}\vec{r_k}(t)$ experiences a break. Only the second derivative of E [see (1)] is still discontinuous, whereas the first derivative is continuous. This follows from the fact that the summation (1) is extended over all atoms, phases of two colliding atoms at the moment of impact are equal and the law of impulse conservation holds: $\Delta_k + \Delta_e = \Delta_k' + \Delta_e'$, where $\Delta_k, \Delta_e, \Delta_k', \Delta_e'$ are the values of Δ, for the k-th and e-th atoms before and respectively after their collision. The break of the second derivative of the expression $e^{i\vec{q}\vec{r_k}} + e^{i\vec{q}\vec{r_e}}$ equals $W = 2(\Delta_k\Delta_e - \Delta_k'\Delta_e')\,e^{i\vec{q}\vec{r_k}} = (\Delta_k^2 + \Delta_e^2 - \Delta_k'^2 - \Delta_e'^2)\,e^{i\vec{q}\vec{r_k}}$, as at the moment of impact $\vec{r_k} = \vec{r_e}$.

On the basis of the well-known theorem of the order of Fourier series coefficients (see Privalov, Fourier Series, § 18) for $\Omega \gg b$, $E(\Omega) = \frac{C}{\Omega^3}\sum_\alpha W_\alpha e^{i\vec{q}\vec{r_\alpha}}$, where summation is extended over all impacts occurring during T, and $\vec{r_\alpha}$ is the radius-vector of the place where the α-th impact occurs. Provided the length of the free path

$$l \gg \lambda/2\sin\frac{\theta}{2} \tag{7}$$

the emission associated with different impacts is incoherent, for the products $e^{i\vec{q}\vec{r_k}}e^{i\vec{q}\vec{r_e}}$ may be neglected. Therefore, assuming (7) we obtain

$$\underset{\Omega \gg b}{Y(\Omega)} = C\frac{\overline{(\Delta_k\Delta_e - \Delta_k'\Delta_e')^2}\,Z}{\Omega^6} \tag{8}$$

where Z is the number of impacts in the scattering volume per sec. and the averaging is performed for various impacts. The value of $\overline{(\Delta_k\Delta_e - \Delta_k'\Delta_e')^2}$ is $\sim 10^{-2} - 10^{-3}b^4$. The calculation of $Y(\Omega)$ for any Ω presents certain difficulties. For $\Omega < b$ the formula (5) [if the condition (7)

* A similar effect occurs also in the case of emission lines, but here it is always superimposed by Lorentz broadening ([6]).

holds] may serve as a roughly approximate expression for $Y(\Omega)$. If for C in (5) is taken the same value as in (8), then

$$Y(\Omega) \underset{\Omega<b}{=} C \frac{N\sqrt{\pi}}{2b} e^{-\Omega^2/b^2} \qquad (9)$$

where N is the number of atoms in a scattering volume.

It is seen from (8) that the intensity of wings accompanying the Rayleigh lines grows $\sim Z = \frac{N}{2\tau_0}$, i. e. proportional to the square of pressure. Applicability of formula (8) is limited by condition (7), the formula being valid only at subatmospheric pressures. One is liable to encounter serious difficulties in finding $Y(\Omega)$ in the regions $l \sim \lambda$ and $l \ll \lambda$ $\left(\text{for } \theta \sim \frac{\pi}{2}\right)$ on the basis of the molecular theory.

In a phenomenological calculation of $Y(\Omega)$ an important rôle is played by the time dependence of the space Fourier component of the density fluctuation, to which the wave length $\Lambda = \lambda/2 \sin\frac{\theta}{2}$ corresponds. Leontowich ([8],[9]) has shown that in order to determine $Y(\Omega)$ it will be sufficient to know the solution of phenomenological equations for the increase in density $\Delta\rho$. These equations are the equations of hydrodynamics, provided the latter can be applied to the sound of the wave length Λ, i. e. to a gas if $\Lambda \gg l$*. In this case ([8]) **

$$Y(\Omega) = C \frac{(\Delta'^2 + \delta^2)\,\delta^2}{(\Omega^2 - \Delta'^2 + \delta^2)^2 + 4\delta^2\Delta'^2} \qquad (10)$$

where $\Delta' = \frac{2\pi}{\Lambda} a'$ is the frequency of sound, a' — its velocity, $\delta = a'\alpha'$ and α' — coefficient of absorption. In the case of a gas, within the limits of applicability of hydrodynamics, $\delta^2/\Delta'^2 \ll 1$ and therefore $\Delta'^2 \cong \Delta^2 = a^2\left(\frac{2\pi}{\Lambda}\right)^2$ and $a' \cong a$, where a is the Laplace velocity of sound; here $\alpha \sim \frac{\Delta^2\eta}{a^3\rho}$ ([9]) (η is the coefficient of viscosity, ρ is the density). We see, therefore, that $\delta = \frac{C}{P}$. When the pressure p is decreased and, on passing into the region $\Lambda \sim l$, a marked drop of Δ' is observed, δ increases and the splitting of the Rayleigh line disappears. If $\Lambda \gg l$ and in a liquid, $\Delta \sim 10^{10}$ sec.$^{-1}$. Therefore $Y(\Omega) = \frac{1}{10\,000} Y(\Delta)$ for $\Omega \sim 10\Delta$, i. e. at distances from the centre of the line smaller than 1 Å. From this it follows that in a gas, as well as in a liquid (if in the latter the fine structure is resolved) at distances of ~ 1 Å from an unshifted line, wings of the line, scattered by density fluctuations, are vanishingly small and may be neglected.

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* Otherwise the fundamental equation of the kinetic theory is to be employed.

** We do not take into account the isobaric fluctuations of density ([10]), whose scattering is practically not broadened.

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PHYSICS

ISOMERISM INDUCED IN In^{115} BY ELECTRON IMPACT

By M. I. KORSUNSKY, V. E. GANENKO and S. I. ZYPKIN

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After the experiments of Collins and co-workers (1), who had found disintegration of beryllium by electrons, and the earlier studies on photodisintegration and photoexcitation of nuclei, the idea was suggested that it should also be possible to excite nuclei with electrons. Beside being an entirely new way of exciting nuclei, this process would be of no less interest as a means to ascertain the mechanism of interaction of electrons and nuclei. The object of the present investigation was to find out whether it is possible to produce nuclear excitation by electron impact.

It is known that excitation can be most conveniently observed, if the nucleus has a metastable state, to which it pass from a higher level excited directly. For our experiments we have chosen In^{115}, since its nucleus is known from earlier investigations to possess a metastable state. As an electron source we made use of a discharge tube fed from an impulse generator. In our first experiments run in June 1939, while working with the entire electron beam formed in the tube, we met a number of grave difficulties; these consisted, in the first place, in that the electron distribution as to energies proved to vary sharply for each impact; moreover, owing to the great heat loading developed in a very short time, the indium simply evaporated.

In order to get rid of these difficulties a monochromator was built. Monochromatization was attained by means of a 90° bend of the electron beam in a homogeneous magnetic field. The accuracy of monochromatization was $\pm 10\%$. The monochromatized electrons got into a Faraday cylinder, at the bottom of which a target with the sample of indium bombarded was placed. This went to eliminate the main difficulty confronted by us in our previous experiments, involved in the determination of the number of active electrons; besides, since all the electrons with energies differing from that let through by the monochromator did no more hit the indium, the second difficulty due to heating of indium was also eliminated.

The target subjected to bombardment consisted of 2 indium foils 0.006 cm thick, separated by an aluminium plate 0.055 cm thick. Thus, the second indium foil was acted upon only by gamma-rays, while the first foil was exposed both to gamma-rays and electrons. The absorption of gamma-rays by Al could be neglected. The difference in the activities excited in the first and second foils and measured with the aid of counter produced a purely electronic effect. In order to reduce the effect of the scattered gamma-rays

formed by electrons previous to monochromatization and in the walls of the monochromator, the end of the latter was lined with aluminium, while the Faraday cylinder was covered with a thick layer of lead. According to Collins' data ([3]), In^{115} has a threshold of excitation at about 1.2 Mev. However, since a considerable loss in intensity of the electron beam took place in the monochromator, bombardment was made by monochromatized electrons with energies ~ 1.6 Mev (at the present time a new, more accomplished monochromator is being designed, which is expected to reduce appreciably the losses in intensity). The results of three independent sets of experiments are summed up in following way: $\Delta_1 = 394 \pm 79$, $\Delta_2 = 34 \pm 77$, $\Delta_3 = 360 \pm 79$. Here Δ_1 is the difference between the activity of the first indium foil and the background of the counter; Δ_2 is the same for the second indium foil; Δ_3 is the difference between the two foils, which produces the direct electron effect.

These data obviously go to confirm the existence of an electron excitation. Owing to the small thickness of the indium foils, the effect from gamma-rays is within statistical error.

The effective cross-section for the electron effect was evaluated according to experimental conditions. The value obtained is $\sim 2 \cdot 10^{-33}$ cm^2. This value is true with an accuracy up to the factor three.

In the course of our last experiments we became acquainted with a note by Collins ([4]) who had been working with electrons of energies equal to 1.3 Mev and obtained results similar to ours (cross-sections 10^{-32} to 10^{-33} cm^2). It should, however, be noted that the results of the present investigation are more precise, since we have been successful in eliminating the effect from gamma-rays. In Collins' experiments this effect was quite appreciable; besides, it has been estimated without taking into account the fact that the conditions of gamma-ray formation in Collins' experiments are similar to those created by a thick anticathode, both sides of which offer different conditions for the formation of excited nuclei.

Before we conclude, it is worth pointing out that there exists no adequate theory of electron excitation admitting of a comparison with the experiment. The formulae for effective cross-section deduced by Wick ([5]) contain matrix elements of intranuclear transitions, magnitudes we still lack sufficient data to evaluate.

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ASTROPHYSICS

ON THE STABILITY OF EMDEN'S COMPRESSIBLE HETEROGENEOUS SPHERES.

By A. B. SEVERNY

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In the present work we are concerned with the stability of hydrostatic equilibrium for a sphere of continuous compressible matter within a field of forces set up by its own gravitation only. It is assumed that the material of the sphere emits no radiation. If ρ is density and p pressure at a distance r from the centre of sphere, then the internal structure of the sphere is determined by the equation

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dp}{dr}\right) = -4\pi G\rho \qquad (1)$$

provided that pressure is connected with density according to the known law. Whether our spheres are composed of a perfect gas with throughout constant internal specific heat (polytropic Emden spheres) or not, the most comprehensive law will be as follows:

$$p = K\rho^{\gamma} \qquad (2)$$

where K is a certain constant and γ a constant characterizing the compressibility of the matter of the sphere. The structure of spheres in equilibrium obeying law (2) was studied by Emden (1) under plausible initial conditions, that at the centre of the sphere the density is finite and at its maximum.

We investigate the stability of these Emden spheres by an approximate method based on variational equations which are obtained from a consideration of small motion near the state of equilibrium, when in the respective equations only terms of the first order with regard to perturbations and their derivatives are retained. The stability we study is a conditional one with regard to perturbations, subjected to the condition that the structure of the sphere retains its spherical symmetry.

The fundamental variational equation for small radial motions of the sphere in the vicinity of equilibrium is the so-called «pulsation equation»(2)

$$\frac{\partial^2\eta}{\partial r^2} + \left(\frac{4}{r} - \frac{g\rho}{p}\right)\frac{\partial\eta}{\partial r} + \left(\frac{4}{\gamma} - 3\right)\frac{g\rho}{rp}\eta = \frac{\rho}{\gamma p}\frac{\partial^2\eta}{\partial t^2} \qquad (3)$$

where

$$\eta = \frac{\xi_1}{r} \qquad (4)$$

and ξ_1 is the displacement of the particle from the state of equilibrium. This equation admits of a solution in the form $e^{i\sqrt{\lambda}t}A$, where A depends on r only and λ is a parameter, so that (3) can be written as

$$\frac{d^2A}{dr^2}+\left(\frac{4}{r}-\frac{g\rho}{p}\right)\frac{dA}{dr}+\left(\frac{4}{\gamma}-3\right)\frac{g\rho}{rp}A=-\lambda\frac{\rho}{\gamma p}A. \tag{5}$$

Here ρ, p and g are certain functions of r, which are given by the solution of the equilibrium problem. Let us now turn to the Emden spheres; it appears expedient to introduce Emden's non-dimensional variables ξ and θ, which is given by means of relations

$$\left.\begin{aligned} &r=\left[\frac{K(n+1)}{4\pi G}\rho_c^{\frac{1-n}{n}}\right]^{\frac{1}{2}}\xi,\ \rho=\rho_c\theta^n,\ p=K\rho_c^{\frac{n+1}{n}}\theta^{n+1}\\ &g=[4\pi GK(n+1)]^{\frac{1}{2}}\rho_c^{\frac{n+1}{2n}}(-\theta'),\ n=\frac{1}{\gamma-1},\ (n>0) \end{aligned}\right\} \tag{6}$$

where $\theta'=\frac{d\theta}{d\xi}$, ρ_c is density at the centre and n «index of the polytrope» (increasing with compressibility of the matter). With these variables equation (1) becomes the Emden equation

$$\frac{d^2\theta}{d\xi^2}+\frac{2}{\xi}\frac{d\theta}{d\xi}=-\theta^n. \tag{7}$$

Solutions of (7) for different n (from 0 to ∞) have been calculated by Emden under the initial conditions referred to above, which, in terms of ξ and θ, will be $\theta=1$, $\theta'=0$ at $\xi=0$.

Introducing Emden's variables (6) into equation (5) and reducing the obtained equation to the normal form with the help of substitution

$$A=\frac{1}{\xi^2\theta^{\frac{n+1}{2}}}u \tag{8}$$

we obtain, owing to (7), the following equation for u:

$$\frac{d^2u}{d\xi^2}-r'u+\lambda'\frac{1}{\theta}u=0,\ \lambda'\equiv\frac{n\lambda}{4\pi G\rho_c} \tag{9}$$

where

$$r'=\frac{2}{\xi^2}-2(n-1)\left(-\frac{\theta'}{\xi\theta}\right)+\frac{n^2-1}{4}\frac{\theta'^2}{\theta^2}-\frac{n+1}{2}\theta^{n-1}. \tag{10}$$

In conformity with the given problem for small motions near the equilibrium, we demand that A should be finite everywhere inside the sphere and at its boundary, whence, in accordance with (4), (6) and (8), function u must necessarily satisfy the following boundary conditions

$$u(0)=u(\xi_0)=0 \tag{11}$$

where ξ_0 is the first root of θ fixing the radius of the Emden sphere. In this way we arrive at Sturm-Liouwille's boundary problem (9), (11). It may be shown that the problem possesses a discrete spectrum of characteristic number λ_k and corresponding characteristic functions u_k, forming an orthogonal set of functions. Our object is to investigate the characteristic number λ_k, or to determine the sign of the quadratic integral

$$D[u]=\int_0^{\xi_0}(u'^2+r'u^2)\,d\xi \tag{12}$$

with condition

$$H[u]=\int_0^{\xi_0}\frac{1}{\theta}u^2d\xi=1. \tag{13}$$

In order that the quadratic form $D[u]$ should have a definitely positive sign, it is necessary and sufficient that in a closed interval $(0, \xi_0)$ there should exist no points conjugated to point $\xi = 0$; in other words, it is necessary that the non-trivial solution of Jacobi's equation for the variational problem (12), (13)

$$\frac{d^2u}{d\xi^2} - r'u = 0, \; u(0) = 0 \tag{14}$$

should not oscillate in the interval $(0 \leqslant \xi \leqslant \xi_0)$ (3). From the viewpoint of this condition, let us now consider Emden spheres of varying compressibilities increasing from $n = 0$ to $n = \infty$.

1. Sphere $n = 0$, $\gamma = \infty$, $\xi_0 = \sqrt{6}$; this is the sphere of an incompressible liquid and its stability has been proved by Liapunov.

2. Sphere $n = 1/2$, $\gamma = 3$, $\xi_0 = 2.7528$. With the help of Sturm's transformation

$$v = \sqrt[4]{\theta}\, u, \; x = \sqrt{n} \int_0^{\xi} \frac{d\xi}{\sqrt{\theta}}$$

the variational problem (12), (13) can be reduced to another analogous problem, for which the form $D[u]$ is definitely positive. Hence sphere $n = 1/2$ is stable.

3. Sphere $n = 1$, $\gamma = 2$, $\xi_0 = \pi$; in this case according to (10) $r' = \frac{2}{\xi^2} - 1$ and the solution of (14) will be $u = \sqrt{\xi}\, J_{3/2}(\xi)$ where $J_{3/2}$ is Bessel's function of the order of 3/2. The least zero u is $4.522 \ldots$, while $\xi = \pi$. Consequently, sphere $n = 1$ is stable.

4. Sphere $n = 3/2$, $\gamma = 5/3$, $\xi_0 = 3.6571$. In conformity with Sturm's theory, the solution of

$$\frac{d^2u}{d\xi^2} - r_m u = 0, \quad u(0) = 0 \tag{15}$$

where r_m is the minorant of r' for interval $(0, \xi_0)$, is found to oscillate more rapidly than solution of (14). Calculation of r' with the help of Emden's tables shows that the first point of the solution of (15) conjugated to zero is $\xi = 4.56\ldots$ Consequently, sphere $n = 3/2$ is stable.

5. Sphere $n = 3$, $\gamma = 4/3$, $\xi_0 = 6.9011$. From (5) it is at once evident that the solution of (14) is: $u = \xi^2 \theta^2$.

Accordingly the point conjugated to zero falls exactly on the boundary of sphere ξ_0 $[\theta(\xi_0) = 0]$; thus, the least characteristic number is zero, and sphere $n = 3$ is in neutral equilibrium.

6. Sphere $n = 5$, $\gamma = \frac{6}{5}$, $\xi_0 = \infty$. Here, beginning from a certain $\xi \geqslant \xi^*$, function $r' \approx -\frac{6}{\xi^2}$. Hence, according to Kneser's theorem, the solution of Jacobi's equation (14) has an infinite set of roots $(0, \infty)$; and sphere $n = 5$ is therefore unstable.

7. Sphere $n = \infty$, $\gamma = 1$, $\xi_0 = \infty$ (an isothermic sphere). In this case a special consideration is required. Omitting detailed computations, we give out the result directly. In this case, Jacobi's equation is also in the form of (14) and function r' beginning from a certain $\xi \geqslant \xi^*$ changes as $-\frac{2}{\xi^2}$; consequently, sphere $n = \infty$ is unstable which was shown by us previously (2).

Conclusions. 1) With increasing compressibility of the matter of the sphere (n rising), the store of its stability (the least characteristic

number) diminishes. 2) Emden's spheres with $n<3$ are stable as regards radial perturbations. 3) Emden's spheres with $n>3$ are unstable. 4) The Emden sphere $n=3$ is in a state of neutral equilibrium and corresponds to the point of bifurcation on the linear series of Emden's spheres. 5) Since for any star built according to Eddington's model and performing small motions near the equilibrium, the effective value of $\gamma > \frac{4}{3}$, it follows that any such star will be stable with respect to radial perturbations.

In conclusion it is to be noted that the condition of instability $\gamma < \frac{4}{3}$ has been repeatedly suggested by various authors, being based on qualitative considerations which were, however, in no way connected with actual investigations of the stability of equilibrium states (be this but in the first approximation). An exception to this is found only in the recent work of Tolman[4]) who, having admitted the applicability of Lagrange's principle to compressible masses, obtained somewhat less full information on the stability of Emden's spheres relatively to radial perturbances; the results arrived at by the author agree perfectly with and are a consistent addition to Tolman's results, even if his starting points are essentially different. The meaning of this difference with reference to the problem under consideration will be given in a forthcoming paper.

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GEOPHYSIK

DIE SONNE UND DIE WECHSEL DER SYNOPTISCHEN PROZESSE IN DEN ÜBERGANGSPERIODEN DER JAHRESZEITEN

Von B. M. RUBASHEV

(Vorgelegt von B. G. Fessenkoff, Mitglied der Akademie, den 27. X. 1940)

Die Gesetzlichkeiten, die durch die gegenwärtigen dynamisch-klimatischen Nachforschungen festgestellt worden sind, gestatten mit ziemlicher Genauigkeit das Datum des synoptischen Wechsels aus einer Jahreszeit in die andere zu fixieren. Dieser Wechsel wird durch eine genügend schroffe Veränderung der Art der früher beobachteten Prozesse charakterisiert: war z. B. in der vorhergehenden Jahreszeit die vorherrschende Art des synoptischen Prozesses die Übertragung der Luftmassen in der Richtung der Breitengrade, und sollte für die kommende Jahreszeit die meridionale Übertragung charakteristisch sein, so wird sich der Wechsel eben durch den mehr oder weniger plötzlichen Übergang zu dem letzten charakterisieren. Sollte aber für die gegebene Jahreszeit die Übertragung der Luftmassen in der Richtung vom Westen nach Osten typisch sein, und für die nächstfolgende die Aufhebung dieser Übertragung auf Rechnung der Umstellung der Antizyklone in die Richtung der sogenannten ultrapolaren Achsen charakteristisch sein, so wird sich der Wechsel gerade durch den Übergang zu den Prozessen, die die west-ostliche Zirkulation vernichten, offenbaren.

Im Lichte der früher durchgeführten Forschungen zur Aufklärung des Zusammenhanges der Impulse der Sonnenaktivität mit den Megaprozessen in der Troposphäre ([1],[2]), schien es wohl möglich, dass auch diese Wechsel mit denselben Impulsen im Zusammenhange stehen. Um diesen Vermutungen nachzugehen, wurden die Daten der Wechsel vom Sommer zum Herbst (28 Fälle) und vom Herbst zum Winter (50 Fälle), Angaben worüber dem Verfasser liebenswürdig von N. A. Schirkina zur Verfügung gestellt waren, einer Durcharbeitung nach der Darstellungsmethode ([1], [3]) unterzogen. Die Ergebnisse sind in den Tabellen 1 und 2 angeführt.

Tabelle 1

27-tägige Phasen des Wechsels zum Herbst (Verspätung 0 Tage) und zum Winter (Verspätung 3 Tage)	Ordinaten der Kurve der ersten Art in %	
	Herbst	Winter
−3	13	11
−2	15	13
−1	22	16
0	23	19
+1	17	13
+2	12	11

Tabelle 2

Phasen der Impulse	Zahl der Wechsel für die gegebene Phase	
	Herbst	Winter
−2	1	3
−1	2	3
0	6	7
+1	5	8
+2	2	5
+3	3	3

Die Durchsicht dieser Tabellen gibt Anlass zu nachstehenden Folgerungen.

1. Aus Tabelle 1 folgt fraglos, dass sowie die Wechsel zum Herbst, so auch zum Winter, unmittelbar mit den Impulsen der Sonnenaktivität in Zusammenhang stehen, denn würde man, den Angaben der Tabelle gemäss, eine Darstellungskurve der ersten Art aufbauen, so würde man sich leicht in ihrer sehr grossen Ähnlichkeit mit der Kurve der Entwicklung des typischen Impulses der Sonnenaktivität überzeugen können.

2. Dieser Schluss wird auch von der Tabelle 2 bestätigt, deren Angaben darauf hinweisen, dass die Phasen der Impulse, auf die sich die meisten Wechsel konzentrieren, sich gerade als diejenigen Phasen erweisen, deren Geoaktivität durch vorgehende Nachforschungen festgestellt worden war.

3. Beachtungswert ist auch die Tatsache, dass die Wechsel zum Herbst mit dem Tage übereinstimmen, an dem die massgebenden Flecken des gegebenen aktiven Gebietes der Sonne durch ihren zentralen Meridian gehen; dagegen verspäten diesem Momente gegenüber die Wechsel zum Winter um drei Tage. Eine erschöpfende Erklärung dieser Erscheinung ist heute wohl schwer möglich. Wenn auch nicht vollständig, so wollen wir dennoch sie zu deuten versuchen, indem wir von folgenden Überlegungen ausgehen:

a) Der Zustand der Troposphäre zur Zeit des Wechsels zum Herbst (meistens im August) ist grundverschieden von dem Zustande beim Wechsel zum Winter (meistens im Dezember). Im besonderen ist die Konvektion im ersten Falle um vieles durchgreifender. Es ist nicht ausgeschlossen, dass dieser Umstand den Prozess zu beschleunigen im Stande ist.

b) Zur Zeit der Wechsel zum Herbst projiziert sich die Erde in höhere heliographische Breiten, als zur Zeit der Wechsel zum Winter. Wie schon bemerkt war, finden in der Tat die Wechsel zum Herbst meistens im August statt. Zu dieser Zeit ist die heliographische Breite des Zentrums der sichtbaren Sonnenscheibe gleich 7°. Die Wechsel zum Winter dagegen finden in der Zeit der winterlichen Sonnenwende statt, wenn die heliographische Breite des Sonnenzentrums in der Nähe von 0° ist. Wie bekannt, erweisen sich aber die äquatorialen Breiten der Sonne als der Bereich, den die Sonnenaktivität meidet.

Diese Verschiedenheit der heliographischen Breiten, in die sich die Erde im Herbst und im Winter projiziert, hat zur Folge, dass im Herbst günstigere Bedingungen für die Durchsetzung der Schwingungen der Sonnentätigkeit in die Erderscheinungen vorhanden sind. Dementsprechend sind diese Bedingungen im Frühling günstiger als im Sommer. Die Bestätigung der Wichtigkeit dieses Umstandes kann unter anderem aus der Kurve der Verteilung der geomagnetischen Störungen während der einzelnen Jahresmonate ersehen werden. Scheinbar ist also die vorerwähnte Erscheinung auch im Stande den Prozess der herbstlichen Wechsel im Vergleich zu den winterlichen zu beschleunigen.

c) Noch eine Überlegung kann zuletzt in Betracht gezogen werden. Wie bekannt, ist es bis jetzt noch nicht möglich zu entscheiden, durch Vermittlung welchen Agentes die Einwirkung der Impulse der Sonnenaktivität auf die Prozesse in der Troposphäre sich verwirklicht, d. h. es ist bis jetzt noch nicht möglich festzustellen, ob dieser Agent photoner oder korpuskülarer Natur ist. Es ist möglich, dass die Einwirkung der Sonne wie durch Lichtquanten von bestimmten Periodenzahlen, so auch durch Korpusküle von statten geht. Die Fortpflanzungsgeschwindigkeit der letzten ist kleiner als die der ersten. Andererseits, erhalten genügend hohe Breiten der Erde (und die betrachteten Wechsel hängen fraglos mit den arktischen Wirkungszentren der Troposphäre zusammen) im August unvergleichlich grössere Zahl der Quanten der Sonnenradiation, als im Dezember. Demgegenüber ist die kor-

puskülare Radiation der Sonne, dank der Ablenkung ihres Flusses zu den hohen Breiten durch das erdmagnetische Feld, viel beständiger in Beziehung zu den Jahreszeiten. Daraus kann man folgern, dass verantwortlich für die Wechsel zum Herbst eher der photone Sonnenagent, und für die Wechsel zum Winter—der korpuskülare ist. In diesem Falle werden sich die Wechsel zum Winter—dem Momente des Durchgangs durch ihren zentralen Meridian der massgebenden Flecken des gegebenen Gebietes der Sonnenaktivität gegenüber—mehr verspäten, als die Wechsel zum Herbst, und zwar dank dem, dass, wie schon erwähnt, der korpuskülare Sonnenagent sich langsamer verbreiten muss, als der photone.

Das sind die vorläufigen allgemeinen Schlüsse, die aus der dargelegten Gegenüberstellung der Daten der Wechsel der synoptischen Jahreszeiten mit den Impulsen der Sonnenaktivität gemacht werden können. Ganz selbstverständlich, muss man aber nicht ausser Acht lassen, dass das Material, welches uns zur Verfügung gestanden hat, in keinem Fall als für endgültige Entscheidungen genügend erachtet werden kann (das gilt besonders für die Wechsel zum Herbst, wo es uns nur 28 Fälle durchzusehen gelang). Solche Armut der Population ist nicht ohne Einwirkung auf die Streuung der Darstellungskurven der ersten Art geblieben, deren Ordinaten in der Tabelle 1 gegeben sind. Dennoch weisen solche Tatsachen, wie gute Übereinstimmung der Darstellungskurven der ersten und der zweiten Art und das Entsprechen (im Sinne der Einwirkung auf die Wechsel) der geoaktiven Phasen der Impulse der Sonnenaktivität den Phasen, die mit den arktischen Invasionen ([1]) zusammenhängen, dass man zu den erhaltenen Ergebnissen, obgleich sie vorläufiger Natur sind, dennoch mit gewissem Vertrauen herantreten darf.

Sternwarte Pulkovo.
Unterabteilung des Sonnendienstes.

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GEOPHYSIK

ZUR THEORIE DES TÄGLICHEN TEMPERATURVERLAUFS IN EINER VERMISCHUNGSSCHICHT

Von A. A. DORODNITZYN

(Vorgelegt von N. Kotchin, Mitglied der Akademie, den 22. XI. 1940)

Es sei die auf eine Einheit der Erdoberfläche fallende Menge der strahlenden Energie $w = \frac{\text{kkal}}{\text{m}^2 \cdot \text{sec.}}$ die gegebene Zeitfunktion. Die Lufttemperatur wird durch die (turbulente) Wärmeleitung, die durch Strahlung hervorgerufene Wärmezufuhr und die advektive Zufuhr bestimmt. Bezeichnen wir die Summe der letzten beiden Teile der gesammten Wärmezufuhr durch $c_p \rho f(z, t)$, wo c_p die Wärmekapazität bei beständigem Druck, ρ—die Dichte der Luft, t—die Zeitdauer, z—die Entfernung von der Erdoberfläche bedeutet. Die Differentialgleichung nimmt hier folgende Gestalt an:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + f(z, t)$$

(T—die Temperatur, k—der Wärmeleitungskoeffizient). Die Wärmezufuhr $f(z, t)$ hängt selbstverständlich von der Entfernung z und von der Zeitdauer t ab. Nun besitzt aber ihr von der Advektion abhängiger Teil eine grosse Änderungsperiode (etwa 6—10 Tage) und kann daher für einen Tag als von der Zeit unabhängig betrachtet werden; der Teil, welcher durch Strahlungsvorgänge in der unteren Luftschicht (1—2 km) bestimmt wird, ist vielmal kleiner als $\frac{\partial}{\partial z} k \frac{\partial T}{\partial z}$, wenigstens bei Abwesenheit von Wolken. Davon zeugt die bekannte Tatsache, dass der Tagesverlauf der Temperatur sich im wesentlichen nur im unteren Kilometer bemerkbar macht.

Wäre die Absorption der strahlenden Energie durch die Atmosphäre der Grösse nach mit dem Gliede vergleichbar, welches durch die Wärmeleitfähigkeit bedingt ist, so würde die Temperaturänderung im Lauf des Tages in der ganzen Höhe der Troposphäre wahrnehmbar sein. Ebenso beweist auch eine Schätzung dieser Glieder, dass im unteren Kilometer die Zufuhr durch Strahlung bloss Prozente oder sogar nur Bruchteile eines Hundertstels von der durch die Wärmeleitung bedingten ausmacht. Deshalb kann mit grosser Genauigkeit angenommen werden, dass $f(z, t)$ nur von der Grösse z abhängt.

Suchen wir eine Lösung für die Temperatur in Form der Gleichung:

$$T = T_0(z) + \sum_1^\infty [\tau_{1n}(z) \cos n\omega t + \tau_{2n} \sin n\omega t] = T_0(z) + \tau(z, t)$$

so erhalten wir für $T_0(z)$ [denn $T_0(z)$ ist augenscheinlich die mittlere Tagestemperatur] und für $\tau(z, t)$ die Gleichungen:

$$\frac{d}{dz} k \frac{dT_0}{dz} + f(z) = 0 \tag{1'}$$

$$\frac{d\tau}{dt} = \frac{d}{dz} k \frac{d\tau}{dz}. \tag{1''}$$

Die Lösung der Gleichung (1') ist eine allgemeine Aufgabe der Atmosphärendynamik und soll uns hier nicht beschäftigen. Eine angenäherte Lösung kann z. B. nach der Methode von Prof. I. A. Kiebel (1) erhalten werden: sie ermöglicht es, die Temperaturänderungen oberhalb der Vermischungsschicht für eine geraume Zeitspanne zu bestimmen.

Unsere Aufgabe beschränkt sich auf die Ermittlung des täglichen Temperaturverlaufs, d. h. auf die Lösung der Gleichung (1''). Neben dieser Gleichung muss auch die Wärmeleitfähigkeit des Bodens in Betracht gezogen werden. Nehmen wir sie als beständig (k_1) an, so gelangen wir endgültig zur Lösung des folgenden Systems von Gleichungen:

$$\left.\begin{aligned} \frac{\partial \tau}{\partial t} &= \frac{\partial}{\partial z} k \frac{\partial \tau}{\partial z} \quad \text{bei } z > 0 \\ \frac{\partial \tau_1}{\partial t} &= k_1 \frac{\partial^2 \tau_1}{\partial z^2} \quad \text{bei } z < 0 \end{aligned}\right\} \tag{2}$$

wo τ_1 die Abweichung der Bodentemperatur von ihrem Mittelwert bedeutet.

Grenzbedingungen. Vor allem liegt ein Abklingen des Tagesverlaufs der Temperatur in grosser Höhe, sowie in tiefen Bodenschichten vor; das heisst:

$$\left.\begin{aligned} \tau(z, t) &\to 0 \quad \text{bei } z \to +\infty \\ \tau_1(z, t) &\to 0 \quad \text{bei } z \to -\infty \end{aligned}\right\} \tag{3}$$

Ferner

$$\tau(0, t) = \tau_1(0, t). \tag{4}$$

Wollen wir nun die Bedingung der Wärmebilanz an der Erdoberfläche formulieren. Drückt man die Zufuhr der strahlenden Energie zur Erdoberfläche durch die Formel $w(t) = w_1(t) + w_0$ aus, wo w_0 die mittlere Tageszufuhr bedeutet, so erhält man:

$$(1 - A) w_1(t) = \sigma [T_0(0) + \tau(0, t)]^4 - \sigma T_0^4(0) - \lambda \frac{\partial \tau}{\partial z}\Big|_{z=0} + \lambda_1 \frac{\partial \tau_1}{\partial z}\Big|_{z=0}. \tag{5}$$

Hier ist $\lambda = c_p \rho k$ und $\lambda_1 = c_1 \rho_1 k_1$; ρ — die Dichte der Luft, ρ_1 — die Dichte des Bodens und c_1 — seine Wärmekapazität.

Die Differenz $\sigma [T_0(0) + \tau(0, t)]^4 - \sigma T_0^4(0)$ sei angenähert durch die Formel $\sigma [T_0(0) + \tau(0, t)]^4 - \sigma T_0^4(0) \cong 4\sigma T_0^3(0) \cdot \tau(0, t)$ ersetzt. Diese Vereinfachung ergiebt einen Fehler von höchstens 10%; da aber das Glied $4\sigma T_0^3(0) \cdot \tau(0, t)$ im Vergleich mit der Wärmeleitung nur klein ist, so vermindert sich der resultierende Fehler für die Grenzbedingung (5) auf nicht mehr als 3%. Somit kann die vereinfachte Bedingung (5) in folgender Form dargestellt werden:

$$(1 - A) w_1(t) = \mu \tau(0, t) = \lambda \frac{\partial \tau}{\partial z}\Big|_{z=0} + \lambda_1 \frac{\partial \tau_1}{\partial z}\Big|_{z=0} \tag{6}$$

wo A das Albedo der Erdoberfläche ist, $\mu = 4\sigma T_0^3(0)$.

Wie ersichtlich, besteht unsere Aufgabe nunmehr in dem Auffinden einer periodischen Lösung des Systems (2) für eine eintägige Periode und die Grenzbedingungen (3), (4), (6).

Der Koeffizient der turbulenten Wärmeleitfähigkeit k. Der Koeffizient k ist in der Atmosphäre allgemein bedeuten-

den Schwankungen unterworfen. Seine Änderung trägt im Grunde folgenden Charakter: er ist nahe der Erdoberfläche fast gleich Null; mit der Entfernung wächst er fast proportionell an und strebt im weiteren einem beständigen Werte zu, nimmt aber in grosser Höhe allmählich wieder ab. Nach Angabe von B. Iswekow, lässt sich diese Änderung von k in der Vermischungsschicht sehr gut durch folgende Formel ausdrücken:

$$k = k_0 (1 + \varepsilon - e^{-mz}); \tag{7}$$

wir wählen hierbei für ε einen solchen Wert, dass bei $z = 0$ der Koeffizient k dem molekularen Koeffizienten der Wärmefähigkeit gleich wird. Dies entspricht der Annahme, dass die Erdoberfläche genügend glatt, also vegetationsfrei ist.

Lösung der Gleichungen. Wir suchen eine Lösung in folgender Form:

$$\left.\begin{aligned} \tau &= \sum_1^\infty \tau_{1n}(z) \cos n\omega t + \tau_{2n}(z) \sin n\omega t \\ \tau_1 &= \sum_1^\infty \tau'_{1n}(z) \cos n\omega t + \tau'_{2n}(z) \sin n\omega t \end{aligned}\right\} \tag{8}$$

Dann werden das System (3) und die Grenzbedingungen durch folgenden Komplex von gewöhnlichen Gleichungen ersetzt:

$$\left.\begin{aligned} k_0 \frac{d}{dz}\left[(1 + \varepsilon - e^{-mz}) \frac{d\tau_{1n}}{dz}\right] &= n\omega\tau_{2n} \\ k_0 \frac{d}{dz}\left[(1 + \varepsilon - e^{-mz}) \frac{d\tau_{2n}}{dz}\right] &= -n\omega\tau_{1n} \\ k_1 \frac{d^2\tau'_{1n}}{dz^2} = n\omega\tau'_{2n}, \quad k_1 \frac{d^2\tau_{2n}}{dz^2} &= -n\omega\tau'_{1n} \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} \tau_{1n}(\infty) = \tau_{2n}(\infty) = 0; \quad \tau'_{1n}(-\infty) = \tau'_{2n}(-\infty) = 0 \\ \tau_{1n} = \tau'_{1n}; \quad \tau_{2n} = \tau'_{2n} \text{ bei } z = 0 \\ (1 - A) a_n = \mu\tau_{1n} - \lambda \frac{d\tau_{1n}}{dz}\Big|_{z=0} + \lambda_1 \frac{d\tau'_{1n}}{dz}\Big|_{z=0} \\ (1 - A) b_n = \mu\tau_{2n} - \lambda \frac{d\tau_{2n}}{dz}\Big|_{z=0} + \lambda_1 \frac{d\tau'_{2n}}{dz}\Big|_{z=0} \end{aligned}\right\} \tag{10}$$

a_n und b_n sind hier die Koeffizienten der Auflösung von $w_1(t)$ in eine Fourier'sche Reihe.

Nun setzen wir folgende komplexe Funktionen ein:

$$\tau_n = \tau_{1n} + i\tau_{2n}; \quad \tau'_n = \tau'_{1n} + i\tau'_{2n}. \tag{11}$$

Die Gleichungen (9) nehmen dabei folgende Gestalt an:

$$\left.\begin{aligned} k_0 \frac{d}{dz}\left[(1 + \varepsilon - e^{-mz}) \frac{d\tau_n}{dz}\right] + in\omega\tau_n = 0 \qquad (a) \\ k_1 \frac{d^2\tau'_n}{dz^2} + in\omega\tau'_n = 0 \qquad (b) \end{aligned}\right\} \tag{12}$$

Die Gleichung (12, b) ergiebt beim Integrieren:

$$\tau'_n = c_n \cdot e^{\frac{1-i}{\sqrt{2}} \sqrt{\frac{\omega n}{k_1}}\, z}.$$

Zwecks Lösung der Gleichung (12, a) führen wir eine neue unabhängige Veränderliche ein:

$$x = 1 - (1 + \varepsilon) e^{mz} \tag{13}$$

(hier entspricht dem Punkte $z=0$ ein $x=-\varepsilon$, und $z=\infty$ ein $x=-\infty$). Es ergibt sich dann

$$x(1-x)\frac{d^2\tau_n}{dx^2}+(1-x)\frac{d\tau_n}{dx}-in\delta^2\tau_n=0 \tag{14}$$

wo

$$\delta^2=\frac{\omega}{k_0(1+\varepsilon)m^2}. \tag{15}$$

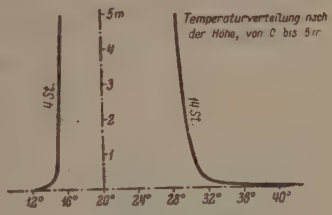
Wir müssen für die Gleichung (14) eine solche Lösung wählen, welche in der Unendlichkeit dem Nullwerte zustrebt. Da diese Gleichung eine hypergeometrische ist, so ist ihre Lösung bekannt, und zwar:

$$\tau_n=(A_n+iB_n)\left(-\frac{1}{x}\right)^{\alpha}F\left(\alpha,\alpha,2\alpha+1,\frac{1}{x}\right) \tag{16}$$

wo

$$\alpha=\frac{1-i}{\sqrt{2}}\delta\sqrt{n}. \tag{17}$$

F bedeutet eine hypergeometrische Reihe; A_n, B_n — vorläufig unbekannte Koeffizienten, deren Werte auf Grund der Grenzbedingungen (10) bestimmt werden müssen. Die Reihe F konvergiert aber nur bei $|x|>1$. Da unsere Grenzbedingungen für $z=0$ aufgestellt sind, also bei $x=-\varepsilon$ (und ε eine sehr geringe Grösse ist), so gilt es eine analytische Fortsetzung der



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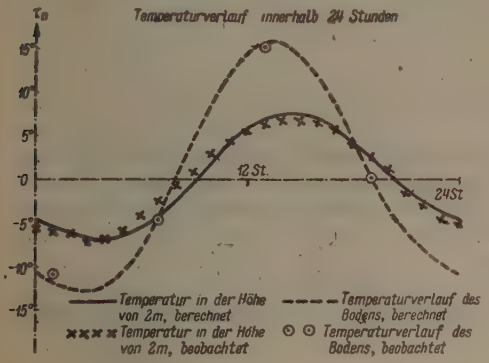

Abb. 1.

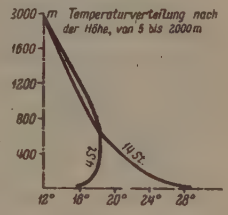

Abb. 3.

Funktion $F\left(\alpha,\alpha,2\alpha+1,\frac{1}{x}\right)$ im Gebiet der Werte $|x|<1$ zu finden. In diesem Zwecke muss man den Barnes'schen Integral anwenden (²):

$$F\left(\alpha,\alpha,2\alpha+1,\frac{1}{x}\right)=\frac{\Gamma(2\alpha+1)}{[\Gamma(\alpha)]^2}\cdot\frac{1}{2\pi i}\int_{-i\infty}^{+i\infty}\frac{[\Gamma(\alpha+s)]^2\Gamma(-s)}{\Gamma(2\alpha+1+s)}\left(-\frac{1}{x}\right)^s ds.$$

Bei Berechnung dieses Integrals mittels der Substraktionstheorie für $|x|<1$ ergiebt sich:

$$\vartheta_n=\left(-\frac{1}{x}\right)^{\alpha}F\left(\alpha,\alpha,2\alpha+1,\frac{1}{x}\right)=\frac{2\Gamma(2\alpha)}{[\Gamma(\alpha)]^2}\left\{-[\ln(-x)\Phi_1(x)-\Phi_3(x)]-\right.$$
$$\left.-\left[2\frac{\Gamma'(\alpha)}{\Gamma(\alpha)}+\frac{1}{\alpha}+2\gamma\right]\Phi_1(x)\right\} \tag{18}$$

wo

$$\Phi_1(x) = F(\alpha, -\alpha, 1, x) \tag{19}$$

$$\left.\begin{aligned} &\Phi_2(x) = \sum_1^\infty c_k x^k, \\ &\text{wobei} \qquad c_1 = -2\frac{a^2}{1!} = 2i\delta^2 n \\ &(k+1)^2 c_{k+1} = \\ &= 2in\delta^2 \frac{(\alpha+1)(\alpha+2)\ldots(\alpha+k-1)(1-\alpha)(2-\alpha)\ldots(k-1-\alpha)}{(k!)^2} \cdot \frac{k-in\delta^2}{k+1} + \\ &\quad + (k^2 + in\delta^2) c_k. \end{aligned}\right\} \tag{20}$$

Es soll nun

$$\vartheta_n = \vartheta_{1n} + i\vartheta_{2n} \tag{21}$$

sein. Dann ist

$$\tau_{1n} = A_n \vartheta_{1n} - B_n \vartheta_{2n}; \quad \tau_{2n} = A_n \vartheta_{2n} + B_n \vartheta_{1n}.$$

Setzt man die Formel (22) in die Grenzbedingungen (10) ein, so erhält man, nach elementarer Umgestaltung, für A_n und B_n folgendes System von Gleichungen:

$$\left.\begin{aligned} (1-A)\, a_n &= A_n \gamma_{1n} - B_n \gamma_{2n} \\ (1-A)\, b_n &= A_n \gamma_{2n} + B_n \gamma_{1n} \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} \gamma_{1n} &= \mu\vartheta_{1n} - \lambda\vartheta'_{1n} + \lambda_1(\vartheta_{1n} + \vartheta_{2n})\sqrt{\frac{\omega n}{2k_1}} \\ \gamma_{2n} &= \mu\vartheta_{2n} - \lambda\vartheta'_{2n} - \lambda_1(\vartheta_{1n} - \vartheta_{2n})\sqrt{\frac{\omega n}{2k_1}}. \end{aligned}\right\} \tag{24}$$

Daraus ergibt sich

$$A_n = (1-A)\frac{a_n\gamma_{1n} + b_n\gamma_{2n}}{\gamma_{1n}^2 + \gamma_{2n}^2}; \quad B_n = (1-A)\frac{b_n\gamma_{1n} - a_n\gamma_{2n}}{\gamma_{1n}^2 + \gamma_{2n}^2}. \tag{25}$$

Zum Schlusse bringen wir ein Beispiel, das sich auf die Stadt Slutzk bezieht. Der 13. VII. 1938 war ein wolkenloser Tag, wie auch der 12 und 14 Juli. Der Zustrom der strahlenden Energie w ist den Angaben des Aktinographen entnommen. Für die Parameter sind Mittelwerte für Slutzk genommen: $k_0 = 8.1$ m²/sec.; $\lambda = 0.5 \cdot 10^{-5}$ kkal/m sec. °C; $m = 0.02$ m^{-1}; $k_1 = 0.9 \cdot 10^{-6}$ m²/sec.; $\lambda_1 = 3 \cdot 10^{-4}$ kkal/m sec. °C; $A = 0.1$.

Die Resultate und ihr Vergleich mit den beobachteten Werten siehe auf unseren Diagrammen.

Eingegangen
den 22. XI. 1940.

ZITIERTE LITERATUR

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Übersetz von A. Tal.

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PHYSICAL CHEMISTRY

APPLICABILITY OF NERNST FORMULAE TO THE CASE OF AMPHOTERIC ADSORBENTS

By A. N. FRUMKIN, Member of the Academy

The problem we are to consider is to find out the conditions, under which the dependence of the charge and potential upon hydrogen ion concentration will obey the same law in the case of an amphoteric non-conductive adsorbent as in that of a hydrogen electrode. Our way to do this is in many respects very similar to that of Lindeström-Lang (1), but differs from the latter by the use of more precisely formulated physical assumptions. Suppose that on a unit surface there are q neutral groups capable to fix hydrogen ions, and q other neutral groups capable to split these ions off. In its initial state the surface is supposed to be free from any hydrogen ion: in this case it carries q negative charges and can obviously fix $2q$ ions of hydrogen. Assume further that the probability for a hydrogen ion to be fixed by any of the $2q$ adsorbing centres would be equal, but for the influence of the electric field created by the surface charges, a factor which can be allowed for if the potential difference between the surface and the bulk of the solution ψ is known. Then the number of hydrogen ions in the unit area of the surface n will be bound with their concentration in the solution $[H^{\cdot}]$ by the following relation:

$$k_1 [H^{\cdot}] (2q - n) = k_2 n e^{\frac{\psi \varepsilon}{kT}} .$$

Hence

$$\psi = \frac{kT}{\varepsilon} \ln \frac{k_1 [H^{\cdot}] (2q - n)}{k_2 n}$$

where k_1 and k_2 are constants, while the rest of the symbols have their usual meaning. Let us now introduce the symbols $[H^{\cdot}]_0$ for such a concentration of hydrogen ions when $n = q$ and $\psi = 0$, that is when the surface becomes electroneutral (iso-ionic point). Then $[H^{\cdot}]_0 = \frac{k_2}{k_1}$ and

$$\psi = \frac{kT}{\varepsilon} \ln \frac{[H^{\cdot}]}{[H^{\cdot}]_0} \frac{2q - n}{n} . \qquad (1)$$

Equation (1) does not give by itself, however, a complete solution of the problem; to find this, we must consider which condition of the surface was taken as the initial one. Let us denote with σ the free charge of a unit surface. We shall have then

$$\sigma = (n - q) \varepsilon \qquad (2)$$

and

$$\psi = \frac{4\pi\sigma\lambda}{D} \qquad (3)$$

where λ is the thickness of the double layer. With $|\psi| \ll \frac{2kT}{\varepsilon}$ the value of λ can be put equal to the thickness of Debye's ionic atmosphere. With $|\psi|$ high enough, λ becomes the thickness of the Helmholtz layer, but we shall not dwell upon that here. From (1) and (2) it follows that

$$\psi = \frac{kT}{\varepsilon} \ln \frac{[H^\cdot]}{[H^\cdot]_0} \frac{1 - \frac{\sigma}{q\varepsilon}}{1 + \frac{\sigma}{q\varepsilon}} \tag{4}$$

If the ratio of the number of surface free charges $\left|\frac{\sigma}{\varepsilon}\right|$ to the highest possible $|q|$ is small, that is $\left|\frac{\sigma}{q\varepsilon}\right| \ll 1$, we can, following Lindeström-Lang, put

$$\frac{1 - \frac{\sigma}{q\varepsilon}}{1 + \frac{\sigma}{q\varepsilon}} \sim e^{-\frac{2\sigma}{q\varepsilon}}. \tag{5}$$

Then

$$\frac{\psi\varepsilon}{kT} + \frac{2\sigma}{q\varepsilon} = \ln \frac{[H^\cdot]}{[H^\cdot]_0}$$

and in view of (3)

$$\frac{\sigma}{\varepsilon} = \frac{1}{\frac{2}{q} + \frac{4\pi\lambda\varepsilon^2}{DqT}} \ln \frac{[H^\cdot]}{[H^\cdot]_0}. \tag{6}$$

Equation (6) is identical to that deduced by Lindeström-Lang. Its use is limited by the condition $\left|\frac{\sigma}{q\varepsilon}\right| \ll 1$, that is

$$\left| \frac{1}{1 + \frac{4\pi\lambda\varepsilon^2}{DkT} \cdot \frac{q}{2}} \ln \frac{[H^\cdot]}{[H^\cdot]_0} \right| \ll 1. \tag{7}$$

Condition (7) is fulfilled in two cases: when

$$\frac{[H]}{[H^\cdot]_0} \sim 1$$

or when

$$\frac{4\pi\lambda\varepsilon^2}{DkT} \cdot \frac{q}{2} \gg 1. \tag{8}$$

In the first case no real analogy exists between the behaviour of our adsorbent and of a conducting electrode, since, with $\frac{[H^\cdot]}{[H^\cdot]_0} \sim 1$, the logarithmic dependence between σ and $\frac{[H^\cdot]}{[H^\cdot]_0}$ can with the same accuracy be replaced by a proportional relation between σ and $\frac{[H^\cdot] - [H^\cdot]_0}{[H^\cdot]_0}$ which is obtained, if no allowance is made for the electrostatic interaction between the adsorbing groups: on the other hand, the coefficient of $\ln \frac{[H^\cdot]}{[H^\cdot]_0}$ in the expression for ψ is not equal to $\frac{kT}{\varepsilon}$.

In the second case it follows from (6) and (3) that

$$\sigma = \frac{DkT}{4\pi\lambda\varepsilon} \ln \frac{[H^\cdot]}{[H^\cdot]_0}$$

$$\psi = \frac{kT}{\varepsilon} \ln \frac{[H^\cdot]}{[H^\cdot]_0}. \tag{9}$$

From (9) it follows that if condition (8) is fulfilled and the number of charges remains small compared to the highest possible one, the potential difference between the adsorbent's surface and the solution is in fact expressed in terms of the concentration, just as in the case of a hydrogen electrode. Condition (8) may also be written thus:

$$\frac{4\pi\lambda\varepsilon q}{D} \gg \frac{2kT}{\varepsilon}. \qquad (10)$$

This means that the limiting potential value which corresponds to the ionization of all neutral groups should be great as compared to $\frac{2kT}{\varepsilon}$.

However, the correctness of these deductions depends also upon how far it is possible to express the action of the electrical field at the surface of the adsorbent through some average potential difference value ψ. To fix this condition precisely is a difficult matter at the present stage of the theory of the double layer([2]). But it is certain, that this conclusion is the more lawful, the smaller the distance between the points on the surface, at which the charges can appear and disappear, as compared to the thickness of the double layer (at the same time we must suppose that the interval, during which the hydrogen ion is exchanged between the adsorbing centre and the solution, is small compared to the time of relaxation of the ionic atmosphere, so that the free charges are uniformly distributed all over the 2 q groups). In a Helmholtz layer, with usual values for interatomic distances and ionic radii, the relation is always inverse; for a diffuse layer in a sufficient state of dilution the condition that

$$q^{-1/2} \ll \lambda \qquad (11)$$

may be realized. The possible use of equation (9) in the case of amphoteric adsorbents is greatly restricted by these considerations. If the potential distribution over the surface can no longer be taken for uniform, there still remains a nearly logarithmic relation between σ and $\frac{[H^\cdot]}{[H^\cdot]_0}$, provided the electrostatic interaction between the different groups is strong enough, but the coefficient of proportionality before $\ln \frac{[H^\cdot]}{[H^\cdot]_0}$ will now take a different value. At last it should be remarked that, irrespective of the electrostatic interaction, the logarithmic dependence between σ and $\frac{[H^\cdot]}{[H^\cdot]_0}$, as follows from M. Temkin's theory, holds good when the adsorbing groups on the surface have different adsorption energies distributed linearly within a sufficiently wide interval of values. In that case the coefficient before $\ln \frac{[H^\cdot]}{[H^\cdot]_0}$ in eq. (9) will, of course, be also different from $\frac{kT}{\varepsilon}$.

Conclusions. It has been shown that the potential difference between the surface of an amphoteric non-conducting adsorbent and the solution can be expressed through the same relation, as is valid in the case of a hydrogen electrode. This is possible when the number of adsorbing groups on the surface is great enough, which is expressed quantitatively by the conditions (10) and (11).

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MINERALOGY

AN ATTEMPT TO REVISE THE CHEMICAL FORMULA OF THE MINERAL HIBSCHITE

By D. S. BELIANKIN, Corresponding Member of the Academy, and

V. P. PETROV

In a previous paper (1) dealing with the mineral hibschite we described a new deposit of this mineral found by us in 1936 in the neighbourhood of the village of Nikorzminda in Georgia, as well as some peculiar local features of this mineral, its paragenesis and constants, being somewhat different from those of the Marienberg hibschite of Cornu. In the present note we give additional results of chemical analyses of the Nikorzminda hibschite.

The material collected at the deposit in 1936 proved insufficient to isolate an amount of hibschite necessary for chemical analysis. Yet a critical analysis of the chemical data reported by Cornu for the Marienberg hibschite showed their deficiency; consequently, the chemical formula of the mineral: $CaO \cdot Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$ derived from these data should also be revised.

In 1939 we revisited Nikorzminda and made the drawing of the hibschite-bearing beds given in Fig. 1; a large supply of the required hibschite material was also collected. The work was facilitated by the fact that the strata particularly rich in hibschite proved likewise to contain a considerable quantity of calcite. In the course of weathering of the rock calcite became partly washed off, and in this way a natural enrichment of the rock with hibschite took place. Besides, the rock became friable, and we had only to rake out the friable arenaceous material from the productive stratum. This friable mass, ground with a wooden pestle to a still finer sand, served as material for our subsequent laboratory work.

A microscopic examination of the sand obtained in this way showed that it was composed of hibschite, garnet calcite and partly of apophyllite. Larger hibschite crystallites invariably contained garnet nuclei, whereas the small fragments of hibschite were more or less monomineral and did not show any intergrowth with garnet. To isolate the purest possible hibschite material, for analysis we used two methods: 1) a partly chemical and 2) a purely mechanical.

In the first case the sand was treated in the cold with 1/10 *N* hydrochloric acid. Thus, calcite was completely solved; evidently, the slightly gelatinized apophyllite was also completely resolved. After washing the garnet-hibschite residue with distilled water to separate the monomineral small fragments from the mixed garnet-hibschite grains, the residue was twice centrifuged with Thoulet's solution, the specific gravity of the solution taken for the 1st and 2nd runs being 3.08 and 3.03, respectively. The hibschite

sample obtained in this way was not quite pure, but still contained some garnet and flakes of silicic acid, produced as a result of decomposition of apophyllite (probably, also of hibschite). The chemical analysis of the concentrates made by V. A. Moleva is given in column 1 of Table 1.

In the second case we used in succession grinding and screening of hibschite sand, until a monomineral state of the sand grains was obtained, these being afterwards centrifuged in a mixture of benzene and methylene iodide with a specific gravity of 3.05 and 3.08. A quantitative mineralogical calculation of this final hibschite concentrate under the microscope gave the following figures: hibschite 90%, garnet 8%, calcite 2%. The chemical analysis of the concentrate was made by I. M. Shumilo; the results are given in column 2 of Table 1.

It is easy to see that both our concentrates, though obtained in different ways, nevertheless closely approach each other with regard to chemical composition, except that the first contains somewhat more SiO_2 (see above—flakes of silicic acid resulting from the decomposition of apophyllite), while the second contains more CaO (from the admixture of calcite). As to the data by Cornu, these show a striking discrepancy with those of the chemical analyses made by Moleva and Shumilo.

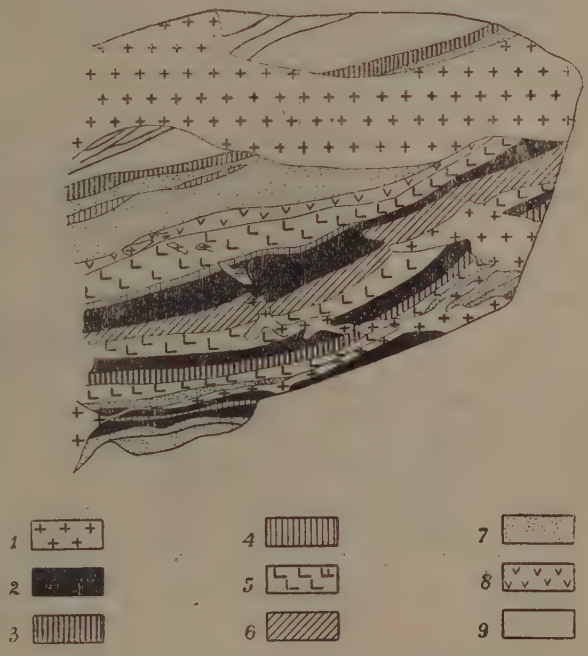

Fig. 1. Scale 1 : 50. *1*—Teschenite. *2*, *3*, *4*—Contact rocks rich in hibschite. *2*—Hibschite-wollastonite. *3*—Hibschite-calcite. *4*—Hibschite-apophyllite (occasionally with calcite). *5*, *6*, *7*, *8*—Contact rocks without hibschite or with negligible quantities of the latter. *5*—Wollastonite-garnet (sometimes with pyroxene and calcite). *6*—Zeolite-wollastonite (with pyroxene, calcite, rarely garnet). *7*—Zeolite-calcite (sometimes with pyroxene or wollastonite, more rarely garnet). *8*—Natrolite-chlorite. *9*—Contact rocks, composition unknown.

Note: 1. Hibschite is always accompanied with garnet, present in the central parts of the octahedrons of hibschite and wollastonite-xonotlite formed at the expense of hibschite. 2. In brackets are given minerals of minor importance in corresponding layers.

After having established the above data, we introduced corrections for admixtures in the composition of our concentrates. The difficulty we encountered consisted in a lack of knowledge of the chemical composition of the most important admixture, garnet. The chemical analysis of this mineral as given by Cornu for the Marienberg case, was anything but satisfactory, being at variance with the optical data: no other data were available. It was necessary to resort to a special isolation of garnet from our materials in order to make chemical analytical determinations. We have chosen a heavy residue (rich in garnet) from the isolation of the second hibschite concentrate from the sand, and subjected it to purification: 1) from the hibschite-calcite

admixture by means of methylene iodide with a specific gravity of 3.34, and 2) from traces of magnetite with a magnet. After this operation a garnet concentrate was obtained which showed under the microscope a hibschite content of 7.5%. Calcite was found to replace garnet in the form of a suspension, and it was impossible to determine it with any degree of accuracy.

Table 1

Chemical Composition of Hibschite Concentrates (1 and 2), the Cornu Hibschite (3) and Garnet Concentrate (4, 5, 6)

	1	2	3	4	5	6
SiO_2 . .	29.10	27.62	37.12	34.68	0.5774	0.5868
TiO_2 . .	0.42	0.65	—	0.75	0.0094	
Al_2O_3 . .	17.69	17.74	32.82	13.21	0.1296	0.2066
Fe_2O_3 .	4.29	4.31	—	12.29	0.0770	
FeO . .	not determ.	0.14	—	none	—	
MnO . .	not determ.	0.08	—	0.06	0.0009	
MgO . .	1.67	1.99	2.64	0.84	0.0208	0.6619
CaO . .	37.03	38.61	13.72	36.00	0.6402	
Na_2O . .	not determ.	none	—	none	—	—
K_2O . .	not determ.	»	—	»	—	—
CO_2 . .	not determ.	1.07*	—	not determ.	—	—
H_2O_+ . .	not determ.	7.70	—	not determ.	—	—
H_2O_- . .	0.48	0.24	13.70	0.24	—	—
Loss on ignition	9.64	8.77	—	2.30	—	—
Total.	100.32	100.15	100.00	100.37		

A chemical analysis of the concentrate was made by Shumilo. The results are given in column 4 of Table 1. After a recalculation of the analytical data to molecular quantities of oxides (Table 1, column 5), the chemical formula of the concentrate: $3.2RO \cdot R_2O_3 \cdot 2.9RO_2$ was obtained.

Thus, we observe here an obvious deviation from the garnet stoichiometry, which is accounted for by the hibschite-calcite admixture in the garnet of our concentrate. If from the 2.30% of the loss on ignition 0.61% be referred to the water of the 7.5% hibschite and the residue—to the carbonic acid of calcite (1.69% CO_2, and the molecular quantity 0.0384), after having deduced an equivalent quantity of CaO (0.0384 mol.) from the total RO, we obtain for RO of the garnet: 0.6619—0.0384=0.6135, and the formula of the garnet will acquire a more normal aspect: $3RO \cdot R_2O_3 \cdot 2.9\ RO_2$. There remains but an insignificant deficiency in RO_2.

To check the accuracy of the chemical analysis made by Shumilo, his figures similarly to the case of the Marienberg garnet of Cornu were recalculated to the elementary garnet components, and on the basis of these components determinations were made, according to an additive formula, of the theoretical specific gravity (D) and the light refraction (N) of our Nikorzminda garnet. These ought to coincide with the values practically measured by us, and this actually happened, as may be seen from Table 2.

If we compare the theoretical constants of the Nikorzminda garnet thus obtained: D=3.65 and N=1.808, with those practically measured, D=3.68 and N=1.808—1.815, we find them to be in good agreement, which goes to confirm the correctness of the analysis made by Shumilo.

The data of this analysis were used for the following recalculation to hibschite of the analytical data given by Shumilo for the second hibschite concentrate. The results of recalculation are given in Table 3.

* By difference: 8.77—7.70.

Table 2

Elementary Constituents and Constants N and D of the Nikorzminda Garnet

Constituents	Weight %	For 100% garnet	Volume % (V)	Spec. gravity (D)	$\frac{D \times V}{100}$	Refract. (N)	$\frac{N \times V}{100}$
$3CaO \cdot Al_2O_3 \cdot 3SiO_2$	46.34	53.25	54.80	3.53	1.94	1 735	0.951
$3CaO \cdot Fe_2O_3 \cdot 3SiO_2$	36.56	42.02	40.71	3.75	1.53	1.895	0.772
$3CaO \cdot Fe_2O_3 \cdot 3TiO_2$	1.67	1.97	1.66	4.30	0.07	2.357	0.039
$3MgO \cdot Al_2O_3 \cdot 3SiO_2$	2.27	2.61	2.70	3.51	0.10	1.800	0.043
$3MnO \cdot Al_2O_3 \cdot 3SiO_2$	0.14	0.15	0.13	4.18	0.01	1.705	0.003
	86.98*	100.00	100.00		3.65		1.808

Table 3

	1	2	3	4	5	6	
SiO_2	27.62	—	2.76	24.86	27.57	0.4590	0.4674—2.02 RO_2
TiO_2	0.65	—	0.05	0.60	0.67	0.0084	
Al_2O_3	17.74	—	1.05	16.69	18.52	0.1817	0.2049—0.89 R_2O_3
Fe_2O_3	4.31	—	0.98	3.33	3.70	0.0232	
FeO	0.14	—	—	0.14	0.15	0.0020	
MnO	0.08	—	0.01	0.07	0.08	0.0011	0.7444—3.22 RO
CaO	38.61	1.12	2.88	34.61	38.39	0.6885	
MgO	1.99	—	0.07	1.92	2.13	0.0528	
H_2O_{-110}	0.24	—	0.02	0.22	0.29	0.0133	
Loss on ignition .	1.07	0.88	0.18	0.01	0.01	—	
H_2O_{+110}	7.70	—	—	7.70	8.54	0.4740	—2.05 H_2O

Note. 1—Weight per cent of oxides in the concentrate. 2—2% of calcite. 3—8% of garnet (from analysis made by Shumilo). 4—90.15% of hibschite substance (difference). 5—The same recalculated to 100%. 6—Molecular quantities.

The molecular proportions of oxides summarized in Table 3 may be expressed more or less satisfactorily by the approximate formula $3\,(Ca, Mg)\,O \cdot (Al, Fe)_2O_3 \cdot 2\,(Si, Ti)\,O_2 \cdot 2H_2O$, which should replace the out-of-date hibschite formula suggested by Cornu: $CaO \cdot Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$. It is of interest to note that the new formula of hibschite closely approaches that of grossularite, except that a part of Al_2O_3 is replaced in it by Fe_2O_3, a part of CaO by magnesia and one particle of silicic acid by two particles of H_2O.

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Translated by W. Gurian.

* Besides, $Al_2O_3 = 0.71$; $CaO = 0.92$; $H_2O = 0.29$; hibschite = 7.66; $CaCO_3 = 3.85$.

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MINERALOGY

ON THE SODIUM SULPHATE WATERS OF THE KUNGURIAN DEPOSITS AT LEVSHINO

By A. M. KUZNETZOV

(Communicated by W. I. Vernadsky, Member of the Academy, 18. X. 1940)

A general picture of the distribution of underground waters of the Kungurian deposits at Levshino has been given by the author in a previous note(1). Waters of four types: 1) the bicarbonate; 2) calcium-sulphate; 3) sodium-sulphate and 4) chloride types may be distinguished in the rocks of the Upper Kungurian underlain by thick beds of anhydrite(2). Great interest as to origin and aggressive properties belongs to the waters of the 3rd type which have been met in different rocks.

The peculiar trait of the Kungurian rock complex consists in that it is rich in gypsum and rather poor in sodium chloride. The composition of the rocks is shown in Table 1.

Table 1

	Number of analyses	Loss on ignition	SiO_2	R_2O_3	CaO	MgO	SO_3	Total
Limestone	60	40.76	5.98	1.87	42.27	3.52	1.10	100.50
Grey dolomite	70	36.44	9.35	3.80	28.80	14.45	7.35	100.19
Clayey dolomite	100	32.79	16.12	7.09	28.86	10.31	3.64	98.81
Marl	94	30.79	19.70	8.92	31.47	6.39	2.25	99.52

In Table 2 are given analyses of waters taken from four bore-holes in different rocks and at a different depth (in g/l).

Table 2

Date of sampling	Well	Depth in m	Dry residue	Na	Ca	Mg	Cl	SO_4	HCO_3
March 5,1937	2675[1]	10.8—15.4	10.638	2.701	0.341	0.280	1.973	4.754	0.177
Do	2353[2]	22.3—24.8	10,980	2.593	0.409	0.285	1.501	5.394	0.122
Sept. 18,1935	2346[3]	35.0—41.0	11.200	2.583	0.520	0.350	1.616	5.703	0.171
March 10,1937	1738[4]	48.0—56.0	10.496	2.491	0.468	0.281	2.068	4.507	0.159

Note. 1—platy limestones; 2—light dolomites; 3—calcareous marls; 4—clayey dolomites. Distance between the wells and the outcrop of the rock at the surface: 2675—1.8 km; 2353—1.3 km; 2346—1.4 km and 1738—1.9 km.

The holes are located in the flood-plain along the line coinciding with the direction of the course of the Kama and the dip of the beds of rocks, which is illustrated by the scheme in Fig. 1. The extreme drill-holes,

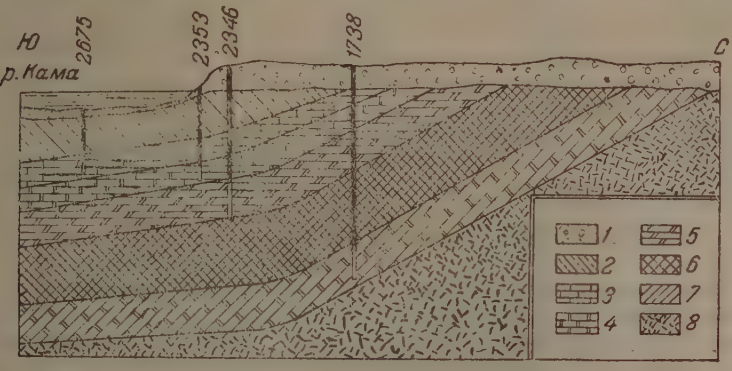

Fig. 1. Schematic geological section. *1*—alluvium; *2*—areno-argillaceous series; *3*—platy limestones; *4*—light dolomites; *5*—calcareous marls; *6*—gypsiferous marls and dolomites; *7*—clayey and banded dolomites; *8*—gypsum and anhydrites. Scale: horizontal 2 km and vertical 20 m in 1 cm.

1738 and 2675, are 1.7 km apart. The distance of each drill-hole from the exposure of the rocks at the surface (under the fluvial drift) differs little, amounting to an average of 1.6 km.

These waters are nearly of the same composition, which may be easily seen from Fig 2, in which the closed curves overlap one another. In the rocks of the region studied there are waters with a content of salts higher than that given above, while the gypsiferous dolomites, lying below calcareous marls and above clayey dolomites, contain chlorine-calcium brines*. This should be taken to mean that the identical waters from different zones have no direct communication. The same is evidenced also by the difference in the composition of the waters: the water from clayey dolomites contains 18—20 mg H_2S which is absent in other waters. The waters contain bromine (5—3 mg) and iodine (1.0—0.1 mg).

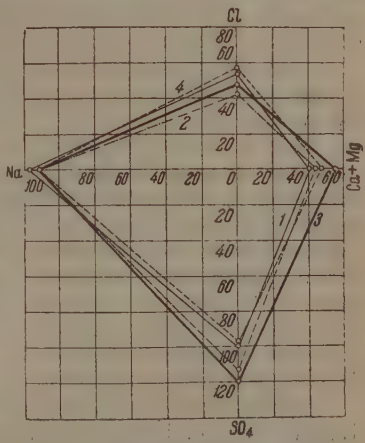

Fig. 2. Waters of the same type in different rocks. Composition shown in milliequivalents per liter. *1*—water of platy limestones; *2*—of light dolomites; *3*—of calcareous marls; *4*—of clayey dolomites.

A considerable accumulation of sodium sulphate has been found in the water from hole 2309 taken from calcareous marls at a depth of 32—40 m; it showed the following composition: salts, 19.23 g/l; $CaCO_3$, 0.16; $CaSO_4$, 1.17; $MgSO_4$, 1.97; Na_2SO_4, 7.70; NaCl, 5.77, and KCl, 2.46. The sulphates in the composition of this water amount to 57%, sodium salts to 70%. The predominance of magnesium atoms over calcium is characteristic.

* For analysis of brine from hole 2622 see p. 256 [1].

The composition of the waters from two wells, the samples being taken at different times (Table 3), varies but little, which testifies to a satisfactory insulation*.

Table 3

Date of sampling	Dry residue in g/l	Ca	Mg	Cl	SO_4	HCO_3
November 15, 1935 [1]	11.970	0.463	0.273	1.514	5.684	0.146
August 4, 1938 [2]	10.370	0.460	0.271	1.095	5.504	0.134
July 2, 1936 [3]	10.880	0.471	0.281	2.024	4.564	0.199
August 10, 1937 [4]	10.360	0.468	0.292	2.108	4.477	0.164

Note. 1 and 2—well 2353, 3 and 4—well 1738.

Well 2353 was subjected during three years to a large number of pumpings, after which the water showed certain decrease in concentration of the chlorine ion, which may be accounted for by the above scheme of water formation. If the pumping of water was continued, the decrease in concentration would probably be more conspicuous.

At the basis of the mineralogy of natural waters, as suggested by W. I. Vernadsky, lies their intimate connexion with the solid matter of the earth's crust, with the gaseous regime and the living matter ([3]). All the waters are therefore considered as heterogeneous mobile equilibria. The existence of underground waters of a homogeneous composition and of a nearly similar mineralization found in marls, dolomites and limestones, points to the similar conditions of their formation. A property common for the rocks, favouring the appearance of such solutions, was their capacity of cationic exchange with natural solutions.

The distribution of the underground waters in the fractured Kungurian formation points to the fact that the movement of the water is accompanied by continuous changes in the salt composition of the solution. The waters entering the rocks are rapidly enriched with calcium sulphate; owing to the comparatively good solubility of the latter, and its high content in the solid phase, the waters remain saturated with this salt throughout their further course**. Further migration of the calcium-sulphate waters involves exchange reactions with the cations adsorbed by the rock. The calcium of the solution, according to the condition of its concentration and the energy of entrance ([4,5]), displaces Na, K, Mg, and the water accumulates sulphates. The presence of sulphates (Na_2SO_4, $MgSO_4$) reduces the solubility of gypsum([6]), while their high solubility makes gradually increase the total mineralization of the water migrating in the rock.

At the same time there proceeds an increase in the concentration of sodium ions by way of leaching of sodium chloride, which results in an increase in the total concentration of sodium ions, and consequently in a suppression of its displacement from dolomite particles; finally, the reaction stops altogether. As soon as the equilibrium concentration of sodium is overreached, the exchange reaction becomes inverse—in a new area of the rock the sodium ion of the solution begins to displace the calcium ion from the solid phase.

* The samples were taken with a bailer after the third drop of the water level in the wells supplied with rubber tampings.

** By leaching experimentally natural gypsum the author has found that when the water moves along a fracture 1 mm wide at an average rate of 0.045 cm/sec., it becomes saturated with calcium sulphate after it has passed 1.5 m. An experiment with leaching of gypsiferous dolomites has shown saturation of the solution with this salt to occur at a lower rate and according to their gypsum content.

The resulting excessive concentration of calcium ions (against the concentration of «gypsum» in the solution), according to the condition of the common ion ([7]), makes the sulphate segregate into the solid phase. In fact as the $CaCl_2$ content in the waters increases, the $CaSO_4$ concentration is reduced. The accumulation of Ca ions in the solution makes it poorer in the sulphate ion, and the water gradually becomes a brine with chlorides of alkaline-earth metals.

The phenomenon of exchange of bases depends on the valency of ions, on atomic weight, ionic radius, concentration, relation of cations, on the accompanying anions, and lastly on the relative volumes of the solid and liquid phases ([4,5,7,3]). The last condition is of particular importance in the existence and migration of underground waters. If the solid phase (silt) of the water basins, where its mass is quite insignificant as compared to the volume of the covering water, has an important effect on the composition of the waters ([9]), in the underground waters, where the solid phase is incommensurably large and is their medium, this condition becomes of paramount importance and can by no means be neglected. If marine silty sediments offer conditions for the accumulation of some element, e. g. K, Br, I ([10, 11]), its concentration in the waters supplied to the rock is greatly facilitated during katagenesis.

Hence it follows that the continuous changes that occur in the composition of natural solutions in the course of their interaction with the solid phase (rocks), as a result of leaching and exchange reactions, bring about the formation of waters of various types showing a regular distribution in the rocks. In fact, in all the lithologically different rocks of the region gypsum-bearing waters have been found situated near the outcrop of the rocks, while bicarbonate waters have been observed at the very exposure. An analogous phenomenon has been observed also in the waters of the Kasan deposits ([12]). In the clayey dolomites chlorine-calcium waters have been met with located South of the belt of the sodium sulphate waters. The presence of waters of the same type in the same rock is due to physical-chemical metamorphism.

The greatest part of the surface of the western Cis-Urals consists of Kazan and Kungurian deposits ([13]). The region of Levshino is hardly the only example of surface waters supplied to the rocks. The possibility is therefore not excluded that a similar formation and distribution of underground waters will be found in some other part of the western slope of the Urals, too.

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MINERALOGY

AXINITE FROM THE MINE «JULIA» OF THE KHAKAS PROVINCE

By S. M. KURBATOV

(Communicated by W. I. Vernadsky, Member of the Academy, 25. X. 1940)

While investigating contact deposits of the province of Khakas in 1931—32, the author encountered in the skarn formations of the mine «Julia» axinite which had been previously unknown in this deposit.

The «Julia» ore deposit is subordinate to a limestone series. The deposit has been opened and worked for copper on the northern slope of the ridge, built up of these limestones, and along its eastern boundary. The limestones of the northern slope possess a marked bedded structure; they are metamorphized as a whole and are also silicified and impregnated with ore excretions to a greater or smaller extent. Separate horizons of these limestones, particularly strongly metamorphized and enriched with ore excretions, represent those ore bodies which were mined. Axinite was encountered by the author in one of these ore beds.

A detailed study of the deposit showed the mineralogical composition and character of all the ore beds to be essentially the same (3). The rock of these beds is usually of a finely stratified texture and consists chiefly of fine-grained very light-coloured diopside, light-coloured pistachio-green calcareo-aluminiferous garnet (Fe_2O_3 content up to 8%) and of the residual calcite. Sphene, vesuvian, epidote, albite and other ore minerals also occur in ore beds.

Axinite was found in such a rock first in one of the dumps and later, after careful searches, on the second horizon of the Russian layer near the mine-surveyor's mark 157. It must be mentioned that the amount of axinite in this deposit is evidently very insignificant as, despite careful searches, only several (about 10) small pieces of ore containing axinite were found. The axinite content in the diopside-garnet rock is also very small. Sometimes axinite forms in the enclosing rock small segregations (up to 6 cm in diameter); more often, however, it gives fine-grained excretions which are responsible together with diopside and garnet for the bedded structure of the rock. Axinite is paragenetically closely related to epidote which is comparatively seldom encountered in the mine «Julia».

No well-developed crystals of axinite were observed. Only granular accumulations of axinite were encountered, the size of individual grains not exceeding 5—8 mm. Axinite possesses a greyish-violet colour.

The study of numerous thin-sections prepared out of metamorphized limestones, taken from various beds at different depths, and also out of

pieces of axinite containing ore, revealed with sufficient precision the succession of formation of minerals and the relative time of formation of axinite. Colourless diopside in the form of small shapeless grains scattered disorderly in the mass of limestone was the first to be formed. Simultaneously or somewhat later took place the formation of very fine crystals of sphene. Next to form was garnet which often encloses diopside grains. The amount of diopside, sphene and garnet varies: sometimes they totally replace limestone and form a compact bedded skarn rock, or else, are scattered all over limestone in a smaller quantity, the remaining space being occupied by the residual calcite; in some cases these minerals are present as separate grains scattered in the mass of marmorized limestone. The detailed general investigation of the mine «Julia» ([3]) and also the study of the vesuvian deposit in the mound 333.5 in the vicinity of the mine «Julia» permits the following conclusion to be made: the diopside-garnet bedded rock formed at the expense of limestone under the influence of SiO_2, TiO_2, Al_2O_3, Fe_2O_3 and MgO brought by hot solutions; diopside, sphene and garnet were formed during the period of the highest temperature. .

Later on, during a colder phase of the hydrothermal process, the secondary alteration of garnet occurred. In the majority of thin-sections it may be observed that garnet was to a considerable extent altered into epidote. Usually an epidote border is seen on the periphery of a garnet crystal; sometimes the process penetrates to a greater depth and affects the entire crystal, the epidote pseudomorphose on garnet being the result. Grains of epidote are very small, shapeless and show no regular orientation.

This fine-grained epidote later underwent recrystallization, and bigger crystals were thus formed. In some thin-sections among the finest crystals of epidote which constitute the pseudomorphose on garnet, there can be observed a small crystal of epidote, possessing already a characteristic elongation and permitting optical constants typical of epidote to be determined. Such a crystal protrudes at times in the process of its growth beyond the pseudomorphized crystal of garnet. In some cases the transfer of the epidote material and formation of separate bigger crystals or small parallel concretions, accumulating in some places of the striated skarn rock, can be observed.

The formation of axinite is confined, perhaps, to this moment; axinite, as it has been pointed out above, is related paragenetically to coarse crystalline excretions of epidote. It is probable that the solution which carries the material for the formation of axinite dissolves the epidote of pseudomorphoses, transfers it and again deposits in the form of bigger crystals together with axinite. Albite was deposited after axinite and encloses grains of the latter. Still later took place the deposition of ore minerals which replace albite and residual calcite.

Therefore, the time of formation of axinite is established rather definitely: axinite was deposited during the hydrothermal phase from not very hot solutions after the formation of epidote at the expense of garnet. A. E. Fersman([6]) in a geochemical diagram of boron confines the formation of axinite in contact skarns to the phases H and I and to the temperature interval of 350—100°. The author of the present paper believes that in the mine «Julia» the formation of axinite began at a temperature much below 350° C.

The following data are the result of the investigation of axinite from the mine «Julia». Very weak, hardly noticeable pleochroism is observed when studying the thin-section on the stage of a microscope with one nicol only. Refractive indices: $Ng=1.682$, $Np=1.673$. Angle of optical axes determined on Fedorov's table $2V=-(70-71°)$. Specific gravity found by the picknometer method equals 3.318 at 4°.

Results of the chemical analysis of axinite carried out by the author are set out in the table. B_2O_3 was determined by T. F. Korzukhina. A recalculation of the analysis given in the same table shows the chemical composition of our axinite to correspond exactly to the formula of axinite established by Groth ([2]) and Schaller ([5]):

$$(Ca, Mn, Fe, Mg, H_2)_7 B_2 (Al, Fe)_4 (SiO_4)_8.$$

Oxides	Weight %	Molecular proportions	Molecular proportions for the types of oxides	Molecular relations	$RO \cdot B_2O_3 \cdot 2SiO_2$	$RO \cdot R_2O_4 \cdot 2SiO_2$	Residue
SiO_2	41.86	0.697	0.697	8.01	0.174	0.348	0.175
Al_2O_3	16.81	0.165	0.174	2	—	0.174	—
Fe_2O_3	1.35	0.009					
B_2O_3	6.09	0.087	0.087	1	0.087	—	—
CaO	20.28	0.362	0.568				
FeO	7.30	0.101					
MnO	3.35	0.047					
MgO	2.34	0.058					
K_2O	0.04	0.001	0.001	7	0.087	0.174	0.348
Na_2O	0.02						
H_2O	0.72	0.040	0.040				
Total	100.16						

The ratio FeO : MnO is nearly 2 : 1 in our axinite. The recalculation of the analysis on R_2O_3 and B_2O_3 to alumoborosilicate (see table) leads to the formula suggested for axinite by W. I. Vernadsky ([1]):

$$CaB_2Si_2O_8 \cdot 2Ca(Al, Fe)_2Si_2O_8 \cdot 2(Ca, Fe, Mn, Mg, H_2)_2SiO_4.$$

It is beyond doubt that the last formula is more correct and represents the nature of axinite. If we consider axinite, after Groth, as an orthosilicate, then the substitution of boron for hydrogen of silicic acid is to be considered probable.

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GÉOLOGIE

SUR LE COMPLEXE SALIFÈRE PERMIEN DU BASSIN DU DONETZ

Par N. YAKOVLEV, membre correspondant de l'Académie

Du cours des dix dernières années, le complexe salifère permien du bassin du Donetz a été traversé par des sondages dans toute son épaisseur, ce qui n'avait pas eu lieu jusqu'ici.

Les deux sondages qui ont traversé le complexe salifère tout entier ont pénétré, le premier (soudière à Slaviansk) jusqu'à la profondeur de 600 m, le second (station Dekonskaïa à l'est de la ville d'Artemovsk) presque jusqu'à 1000 m. Pareilles profondeurs n'ont pas encore été atteintes dans ces régions, et comme le premier sondage a été foré à l'ouest de la dépression de Bakhmout-Slaviansk et le second à l'est, c'est-à-dire à la plus grande distance possible l'un de l'autre, ils sont intéressants tant chacun en particulier que comparés entre eux. Si l'on néglige les faibles indices de la présence du sel au sommet et à la base même du complexe (il vaut mieux n'en pas tenir compte dans l'étude comparative des deux sondages, vu qu'ils ont pu être remarqués par le chef sondeur lors du forage de l'un des sondages et passer inaperçus lors du forage de l'autre), les coupes du sondage No. 6 à Slaviansk et du sondage de Dekonskaïa présentent toutes deux 8 couches de sel pouvant être identifiées les unes aux autres d'après leur position dans le complexe, qui est désignée par un numéro (I—VIII).

Le petit tableau ci-après indique les puissances (en m) des couches de sel et entre parenthèses celles des assises intermédiaires (numération de haut en bas).

	Slaviansk	Dekonskaïa
Couche I...	36.00 (39.00)	32.55 (34.20)
» II...	5.00 (23.50)	8.10 (19.20)
» III...	46.50 (24.50)	37.90 (71.20)
» IV...	16.50 (9.50)	8.00 (3.00)
» V...	33.50 (28.50)	6.00 (9.90)
» VI...	4.00 (8.60)	2.97 (13.61)
» VII...	0.80 (8.10)	1.00 (6.50)
» VIII...	7.50	0.70
Puissance totale	149.80 (141.70)	97.22 (157.61)
Puissance du complexe salifère	291.50	254.83

L'augmentation en puissance du complexe salifère à Slaviansk comparativement à Dekonskaïa est due aux couches de sel, dont la puissance totale est un peu supérieure à la moitié de la puissance du complexe tout entier, alors qu'à Dekonskaïa elle constitue un peu moins des deux cinquièmes de celle-ci.

Après un accroissement brusque de l'intervalle entre deux couches de sel a lieu aussitôt la sédimentation d'une couche de sel plus puissante (couches V, III et I à Slaviansk, III et I à Dekonskaïa).

Il paraît probable que lors du dépôt d'une importante assise de roche stérile, le détroit par lequel le golfe où se déposait le sel communiquait avec la mer, était obstrué par des sédiments terrigènes argileux et le rétrécissement du passage permettait une concentration accrue des eaux salées et le dépôt d'une plus grande quantité de sel.

La formation du sel gemme atteint son intensité maximum dans la partie supérieure du complexe salifère, où sont réunies les couches de sel les plus puissantes. La couche dite de Briantsevka, dont l'exploitation fut commencée avant celle des autres à la mine de Briantsevka, est la 3-e à partir d'en haut. A Briantsevka même elle est la seconde, par suite du coïnçage de la première couche.

Au-dessus de la première couche indiquée dans le tableau ci-dessus se trouve encore une couche de sel de 2 m d'épaisseur avec intercalations d'anhydrite, séparée de la première par une couche d'anhydrite de 16.60 m. Les 21 m supérieurs de la première couche renferment aussi des intercalations d'anhydrite.

La couche de sel supérieure, de 2 m d'épaisseur, est surmontée de 62 m de gypse, d'anhydrite, de grès, de marne et d'argile, qui constituent le complexe gypsifère supérieur, tandis qu'au-dessous du complexe salifère s'étend le complexe gypsifère inférieur. Le complexe gypsifère supérieur est recouvert à son tour du complexe gréso-argileux supérieur, durant la formation duquel il ne s'est déposé dans la dépression de Bakhmout-Slaviansk ni anhydrite, ni sel; c'est alors que le golfe fut privé de communication avec la mer et commença à se combler de sédiments terrigènes.

Intéressante est la découverte dans le sondage de Dekonskaïa et ensuite au puits No. 3 de la mine Liebknecht (ci-devant de Briantsevka), d'une couche de marne dolomitisée à faune de brachiopodes, de foraminifères et d'autres fossiles que l'on trouve aussi dans la faune du complexe calcaréo-dolomitique sous-jacent au complexe salifère. Cette couche de marne (évidemment lenticulaire, puisque, semble-t-il, elle n'affleure pas à la surface dans les autres mines non plus), est située à 11 m au-dessous de la couche I (dite de Briantsevka) dans le sondage de Dekonskaïa et a une grande importance pour la détermination de l'âge du complexe, qui jusqu'ici n'avait presque par fourni de faune fossile. L'identité de sa faune et de celle du complexe calcaréo-dolomitique atteste la continuité dans le temps de la formation des complexes salifère et calcaréo-dolomitique, qui ont le même âge géologique. Les tentatives faites dernièrement de prendre les cailloutis situés un peu au-dessus du complexe dolomitique pour des conglomérats témoignant d'un arrêt dans la sédimentation du Paléozoïque de la dépression de Bakhmout-Slaviansk et d'une transgressivité des couches sous-jacentes au complexe calcaréo-dolomitique sur ce dernier, sont donc sans fondement.

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GEOLOGY

FERRUGINOUS MANGANESE ROCKS IN THE PHOSPHORITE-BEARING BEDS OF THE KARA-TAU

By P. L. BEZRUKOV

(Communicated by A. N. Zavaritsky, Member of the Academy, 9. VII. 1940)

It is known that sedimentary phosphate and manganese rocks are frequently found in close proximity. As an example one may cite the Algonkian deposits of carbonate of manganese in the Ferruginous Mountains (Czechia), individual beds of which are greatly enriched with phosphate. In the Cambrian deposits of Newfoundland carbonate manganese rocks are interbedded with phosphorites. The joint occurrence of phosphate and manganese sediments is observed also in the Ordovician of Arkansas and in many other places. In the USSR a phosphorite-bearing layer has been encountered by the author at the base of Tertiary carbonate manganese ores of the North Urals (1). The examples cited above seem to suggest the idea that the conditions favouring the accumulation of carbonate of manganese and phosphates in sea basins ought to have some features in common.

In 1937 in the course of the geological prospecting work carried out in the new highly phosphorite-bearing basin of Kara-tau (2), the author detected among the rocks of the phosphorite-bearing beds a very interesting horizon of ferruginous manganese carbonate rocks. Subsequent work done in 1938—1939 allowed it to be traced nearly throughout the phosphorite-bearing region.

The phosphorite-bearing beds of Kara-tau are overlying the sandstones and siliceous schists (silicites) of the Karoi series (Lower Cambrian) and are overlain in turn with dolomites and dolomitized limestones of the Tamda series (upper half of the Cambrian—Lower Silurian). After the trilobite fauna collected in the lowermost portion of the Tamda series, the age of the phosphorite-bearing beds is determined as Middle Cambrian (4). At the base of the beds, at their boundary with the Karoi rocks, a small (2—20 m) horison of «lower» dolomites is lying transgressively. The rocks of the phosphorite-bearing beds are separated from the «lower» dolomites also by traces of erosion.

The summarized section of the phosphorite-bearing beds is as follows:

a) Slightly phosphatized, yellowish-grey and black flints (silicates) are lying on the uneven surface of «lower dolomites», containing individual bands of phosphorites and phosphatized dolomites. Thickness 0—25 m.

b) Bedded phosphorites with bands of phosphorite conglomerates, phosphate-siliceous schists, phosphatized flints and dolomites. The layers are frequently composed of phosphorites only. The boundary between the two series is not always sufficiently distinct, occasionally there is observed a

facial cross-transition of the lower part of the productive series of layers to the upper part of the siliceous series. Occasionally series «b» is directly overlying the «lower» dolomites. The total thickness of the phosphorites is 0—35 m; the thickness of the whole packet 0—60 m.

c) Horizon of ferruginous manganese carbonate rocks. Thickness 0—5 m.

d) Slightly phosphatized platy dolomites, at places siliceous, with several thin phosphorite bands. Thickness 0—15 m.

The total thickness of the phosphorite-bearing beds amounts to 75 m.

As a rule, between these beds and the overlying pure non-phosphatized dolomites there are observed gradual transitions.

The phosphorite-bearing beds may be traced in several parallel bands throughout the north-eastern branch of the Kara-tau, over a distance of about 120 km. Over the whole region a zone of maximum phosphate accumulation is distinguished striking in a north-western direction. Both south-west and north-east from this zone the phosphorite-bearing beds become thinner; at places they are completely thinning out. In the south-eastern part of the region the phosphorite-bearing rocks of the Lower Paleozoic are penetrated by Caledonian granites. Near the contact with these the phosphorites are subjected to metamorphism and in some deposits are replaced by apatites(3).

As shown above, a horizon of ferruginous manganese carbonate rocks is located in the upper part of the phosphorite-bearing beds at the boundary of the productive series of beds with the slightly phosphatized carbonate rocks. Its deposition must have coincided with a rather sharp change in the facial conditions; this horizon is overlain only with thin (0.05—0.10 m) bands of phosphorites interbedded between dolomites which only occasionally form a rather thin layer (up to 0.15 m) directly over the manganese rocks.

The petrographic composition of the horizon under examination within the phosphorite-bearing region suffers considerable changes. It is represented sometimes by slightly ferruginous dolomites with individual coatings of manganese oxide, and sometimes by solid ferruginous manganese rocks. The rocks occur in plates with knobby outgrowths at the surface and usually show a thin, at places sinuous lamination. Sometimes the texture of the rocks is pisolitic or else they show no regular texture at all. The rocks under description are brownish-red, pinkish-grey, dark-brown or almost black. Differently coloured layers are frequently interbedded. Reddish tinges are produced by iron oxides, black ones—by pyrolusite forming at places considerable segregations.

Viewed under the microscope, the rocks consist chiefly of finely-crystalline carbonate, as well as of bands, spots and strips of iron and manganese oxides. Sometimes the latter form a solid mass of rock. Aside from these, there are present greater or lesser quantities of silica (chalcedony), colloidal or finely-crystalline phosphate, clayey particles, small segregations of pyrite and, more rarely, quartz grains and minute phosphorite pebbles and grains. The ratio of individual components is not constant, which may be seen from the lamination and facial alteration or the rocks along the strike. In case of a micro-laminated texture, the thickness of individual homogeneous bands amounts but to hundredth of a millimeter. The bands are frequently crumpled, forming microfolds.

A study of the carbonate mass by usual optical methods is prevented by the diminutive size of the crystals. In a number of cases, however, dolomite, rhodochrosite and at places calcite and siderite are distinguished in the mass. Siderite seems to be largely an admixture to rhodochrosite. The interrelation of the carbonates requires special investigation. Iron and manganese oxides, represented chiefly by turgite, haematite and pyrolusite replace usually the carbonate mass. Undoubtedly, they must have preci-

pitated later. Primary rocks evidently consisted chiefly of carbonates of magnesium, calcium, manganese and iron, and it was not until surface weathering that they became enriched with oxides. It should be assumed that the relative amount of carbonates must increase with depth.

In the rocks under description chalcedony most frequently forms thin regular bands, while in pisolitic varieties it forms concentric layers alternating with layers of oxides of iron, manganese and carbonates. It was evidently also deposited at the bottom of the basin by way of precipitation from colloidal solutions.

The chemical composition of the rocks described is as variable as their petrographical composition. In rocks enriched with iron the Fe_2O_3 content is not above 30%, rocks relatively rich in manganese contain up to 39% MnO. The quantity of calcium and magnesium oxides, as well as of SiO_2 varies within a wide range; the P_2O_5 content varies from 1 to 3.8%, the Al_2O_3 content from 3 to 6%; free alumina is absent. Thus, the quality of the ironstone, as well as of the manganese ore is low.

Certain samples of manganese carbonate rocks were subjected to spectral analysis in the Laboratory of the Institute of Geological Sciences of the Academy of Sciences of the USSR. The analyses showed the presence in the rocks of Ni, Co, Mo and Ba in appreciable amounts (medium lines), and Be, Cu, Zn, V, Na and Ti in small quantities (feeble lines).

As shown above, the horizon of ferruginous manganese carbonate rocks is distinguished by a variability of composition and thickness. The greatest thickness is obtained along the margins of the region of development of the phosphorite-bearing beds. The manganese ores are developed at the deposits Tashik-tas, Kyr-Chabakty, Dzhetym-tal, Kara-shat, Berkuty etc.; their thickness here varies from 0.5 to 2.5 m; within the zone of maximum phosphate accumulation (deposits Kok-su, Dzhany-tas, Ak-sai, Chulak-tau) this horizon shows its smallest thickness and at places it is completely thinning out. This circumstance is evidently accounted for by the fact that the material for the formation of the rocks under examination (iron and manganese compounds) was carried in from the dry land and deposited chiefly near the shore.

In the south-eastern part of the region the rocks of the phosphorite-bearing beds are at places penetrated by granite intrusions and subjected to contact metamorphism. Unfortunately, we failed up to the present to trace the alterations in the composition of ferruginous manganese carbonate rocks at their approach to granites. Yet, at the deposit Nasyn-kul, in the manganese ores covering apatite rocks, crystals of chlorite, muscovite, secondary quartz and apatite were detected along with manganese oxides and carbonates.

From the above data it may be seen that the Cambrian deposits of Karatau show a very curious combination of a number of chemical sediments: phosphate, siliceous, ferruginous and manganese. A condition invariably calling forth their accumulation is a break in the supply of terrigene material; as a source of phosphates served sea water, of the others components—waters of continental drainage. The deposition of ferruginous manganese sediments was confined to the moment of replacement of the siliceous and phosphate sediments with calcareous ones.

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STRATIGRAPHY

NEW DATA ON THE STRATIGRAPHY OF THE UPPER CARBONIFEROUS OF THE OKSK-ZNINSK BANK

By D. RAUSER-CHERNOUSSOVA

(Communicated by P. I. Stepanov, Member of the Academy, 26. X. 1940)

Our present knowledge of the stratigraphic disjunction and thickness of the Upper Carboniferous of the Oksk-Zninsk bank has been based up to now only on investigations of N. Sibirzev (2). Subsequent researches (1, 2, 3) and unpublished reports added little new to the views of that investigator. On the other hand, deep boring begun on the Oksk-Zninsk bank requires a number of problems of the stratigraphy of the Carboniferous and particularly of its upper division to be reconsidered.

From this point of view results of boring conducted by the Administration for Construction of the Kuybyshev Water Power Plant on the Oka, in the region of Murom on the eastern slope of the Oksk-Zninsk bank, are of special interest. A number of bore-holes were sunk at a distance of 15 km to the South from Murom. Cores from five bore-holes were handed over to the author of the present paper to be worked up for foraminifers. All these bore-holes (105, 109, 122, 128 and 207) are located at a distance of only 20 km from one another. Notwithstanding the fact that no Middle Carboniferous fauna of foraminifers was found in bore-holes which penetrated to different depth into the upper Carboniferous, and that the microfauna was in no good condition, especially in the upper part of the section, it was still possible to establish an approximate total thickness of the Upper Carboniferous and subdivide it into six horizons.

Triticites Series (C_3^{II}). Horizon with *Triticites montiparus* (C_3^{Ia}). Fissile limestones (in the upper part also dolomites) interbedding with multicoloured marls of violet-red and greenish hues and clays with parting of organogenous clastic and fusuline limestones; more seldom these partings consist of organogenous oozy limestones with small foraminifers. The characteristic forms are: *Triticites montiparus* Moell., *T. umbonoplicatus* Raus. et Bel., *T.* ex gr. *arcticus* Schellw., *Quasifusulina longissima* Moell., *Fusulinella pulchra* Raus. et Bel., *Staffella angulata* Col., *Bradyina natiliformis* Moell. Visible thickness—27 m; total thickness is, probably, about 40—50 m.

Horizon with *Triticites arcticus*, *T. schwageriniformis* (C_3^{Ib}). Higher was encountered a pack of predominating limestones, these being often of organogenous oozy character with small foraminifers and sometimes of fusuline and organogenous clastic nature, with rare and variable partings of dark calcareous clays (bore-hole 207). The following forms were identified: *Triticites schwageriniformis* Raus., *T. arcticus* Schellw., *T. noiskyi* Raus., *Quasifusulina longissima* Moell., *Parastafella* ex gr. *bradyi* Moell., *P. moelleri*, *Globivalvulina* sp. a. This horizon (30—35 m) is characterized by a considerable variability of facies and species composition of foraminifers.

Horizon with *Triticites stuckenbergi* (C_3^{Ic}). At the base of this horizon lies a pack of mottled micaceous clays interbedding with unctuous clays; thickness of the pack in different bore-holes varies from 1 to 4 m. Sometimes there occur arenaceous clays with sandstone partings (bore-holes 128, 207). Still higher is located a thick pack of often dolomitized limestones; less often are observed dolomites with rare and thin partings of clays and marls. Thick (up to 4 m) partings of well-preserved fusuline limestones are encountered quite often. Limestones are facially subject to variations. In the lower part of the horizon prevail organogenous clastic and coprogenous clotted limestones with sponges and small foraminifers. The following complex of foraminifers has been determined: *Triticites stuckenbergi* Raus., *T.* ex gr. *stuckenbergi* Raus., *T.* ex gr. *paraarcticus* Raus., *T. arcticus* Schellw., *T.* ex gr. *volgensis* Raus., *T.*ex gr. *jigulensis* Raus., *T.* ex gr. *ventricosus* Meek., *T.* ex gr. *subventricosus* Ozawa, *Quasifusulina longissima* Moell., *Globivalvulina* sp. a. Thickness of the horizon with *T. stuckenbergi* is about 40 m.

Dolomite Horizon with *Triticites jigulensis* (C_3^{Id}). Stratigraphically higher lies a thick pack of dolomites and dolomitized limestones sometimes silicified, with partings of greenish marls and clays. Dolomitized limestones predominate in the lower 15—20 m, whereas in the upper 30 m prevail dolomites. Total thickness of this horizon is about 45 m. Foraminifers from the horizon under consideration were encountered in the bore-holes 114 and 207 only, although some beds abound in cavities from fusulinides. Fusulinides from the lowermost portion of this horizon were found to be new species of the groups *Triticites* ex gr. *jigulensis*, *T.* ex gr. *volgensis*.

Pseudofusuline Series (C_3^{II}). Higher occurs a pack of dolomites which are often silicified, sometimes decomposed to the state of dolomite farina and contain grey or black (seldom) flints. Dolomites often abound in cavities from fusuline. Corals are found less frequently. Only *Pseudofusulina sokensis* Raus., *P.* ex gr. *sokensis* Raus. were identified among fusulinides. Thickness is found to be 20 m.

Schwagerina Series (C_3^{III}) and Tastuba Horizon (C_3^{IV}). The *Schwagerina* and overlying Shustovo-Denyatin series are lithologically similar and strongly disintegrated; their separation in cores of the bore-holes is impossible because fusulinides are encountered only in the lower portion of the *Schwagerina* series, and when encountered are scarce and poorly preserved. A portion of Shustovo-Denyatin siliceous dolomites and flagstones was supposedly referred by the author of the present paper to the uppermost part of the Upper Carboniferous, i. e. to the Tastuba horizon with *Pseudofusulina moelleri*. In four bore-holes these series are represented chiefly by dolomites (often silicified) and dolomite farina with black flints. The identified fusulinides are as follows: *Pseudoschwagerina muongthensis* Deprat and *Pseudofusulina* ex gr. *paramoelleri* Raus. (bore-hole 128). In descriptions of bore-holes the cavities from *Schwagerina* are indicated for higher parts of the *Schwagerina* horizon as well. Total thickness of this series is about 15—20 m. The finding of *Schwagerina* enabled about 10—15 m to be referred supposedly to the *Schwagerina* horizon and the remaining 5—10 m to the Tastuba horizon.

Although the study of foraminifers from the investigated bore-holes was not complete and had a preliminary character, still we succeded in pointing out six main horizons of the Upper Carboniferous, which are also traced on vast areas in neighbouring regions.

Horizon with *Triticites montiparus* (C_3^{Ia}) is always characteristic of the lower part of the Upper Carboniferous. Species of this group are the mass forms in the *Teguliferina* horizon of the Moscow basin (mostly in the lower

part of the horizon—in the series of red marls), in synchronous deposits of the Northern district (Prionegie), on the Samara Luka (Bend), in Zavolgie (the land on the left bank of the Volga), in Priuralie (Tuimazi) and also in the Donbass.

Horizon (C_3^{Ib}) with *T. arcticus* and *T. schwageriniformis* has been traced up to the present time on a smaller area, namely only to the East of the Oksk-Zninsk bank in Povolgie (the land along the Volga). This horizon is characterized in different regions by a variable composition of the fauna of foraminifers; this fact was also corroborated in the Murom region.

Horizon with *T. stuckenbergi* (C_3^{Ic}) is established with sufficient certitude in Povolgie and Zavolgie. A considerable species variability of the complex of this horizon is a typical feature. The upper horizon with *T. jigulensis* widely extends, evidently, to the East of the Oksk-Zninsk bank, being also well represented in the North (the Kotlass bore-hole).

In higher horizons, in *Pseudofusulina* and *Schwagerina* series, were encountered species having a wide, almost general occurrence: *Pseudoschwagerina muongthensis*, *Pseudofusulina* ex gr. *krotovi*, *P. sokensis*.

Rough comparisons of the lithological composition of the Upper Carboniferous of the Oksk-Zninsk bank with that of regions adjacent to it East and West are of interest.

Depositions of the lower horizon with *P. montiparus* are lithologically very close to the *Teguliferina* precipitations (reddish-violet marls) of the Moscow district, but sharply differ from synchronous depositions of the Samara Luka (Bend) where organogenous clastic, fusuline and alga limestones are developed; less often there occur clotty foraminiferous limestones with frequent signs of silification.

However, already the next horizon C_3^{Ib} bears a close resemblance to the Samara Luka (Bend), the resemblance being increased in higher horizons. Thick packs (up to 4 m) of fusuline alga limestones are especially characteristic of this horizon. Of great interest is the appearance of micaceous clays or arenaceous clays and even sandstones at the base of the horizon C_3^{Ic} on the Oksk-Zninsk bank. Approximately at the same time a phase of the most shallow depositions, expressed in a typical flora and fauna complex, is observed on the Samara Luka (Bend).

It is interesting to note a close analogy in the nature of depositions of the upper part of the horizon C_3^{Id} of the Oksk-Zninsk bank and of the Samara Luka (Bend); this analogy manifests itself in the appearance of greenish dolomite varieties and partings of greenish marls. In general a considerable amount of the argillaceous material is typical of the investigated section nearly as a whole.

The total thickness of the Upper Carboniferous of the Oksk-Zninsk bank was found to be quite large in the area of the platform, namely 185 m; when evaluating the figures obtained, it should be taken into account, on the one hand, that there exists no sharp border with the Middle Carboniferous (this will perhaps lead to an increased thickness of the Upper Carboniferous as the result of subsequent investigations), and on the other hand, the top of the Upper Carboniferous is also indistinct.

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GENETICS

EXPERIMENTAL PROOF OF NON-UNION OF CHROMOSOME FRAGMENTS AT THE FORMATION OF RECIPROCAL TRANSLOCATIONS

By I. B. PANSHIN

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According to the crossing-over hypothesis of the mechanism of formation of chromosome rearrangements advanced by Serebrovsky (1) and Dubinin (2) and sustained and developed by Muller (3) on the basis of the probable reciprocity of all rearrangements, chromosome rearrangement are supposed to be due to non-homologous exchange between chromosome parts caused by their immediate contact. Both breaks taking place in this way are due to one and the same cause. This hypothesis, shown by the reciprocity of all rearrangements and by the absence of chromosomes with simple breaks, explains these important peculiarities of rearrangements by the mechanism of the rearrangement process itself. According to the fragmentation hypothesis of Stadler (4) and Muller (3) independent chromosome fragmentation takes place with subsequent reattachment at any breakage points. The reciprocity of exchanges and the absence of simple breaks ought to be the result of nonviability of the zygotes in the case of non-union of fragments, caused by aneuploidy for the acentric as well as for the centric fragment with a simple break which, as required by the hypothesis, must cause the destruction of this fragment.

A number of papers, which cannot be briefly quoted, devoted to the study of complex rearrangements as well as to a statistical analysis and to a direct cytological study of the process of the occurrence of rearrangements and breaks, speak in favour of the fragmentation hypothesis. The data of McClintock (5, 6) and Sax (7) are especially convincing. According to the crossing-over hypothesis, the chromosomes invariably exchange parts during the occurrence of the rearrangement. From the point of view of the fragmentation mechanism, if chromosome breaks and unions are independent processes, in case of two obligatory breaks, two fragments may join and form a new order, while two others remain unjoined. The experimental reproduction of such cases in ordinary structures is complicate or even impossible, being necessarily accompanied with aneuploidy. However, the cytogenetic proof of the non-union of fragments may be attained by using special structures which permit to utilize the specific properties of the sex chromosomes of *Drosophila*.

An inverted chromosome, consisting of a left part of the w^{mt4} and of the right part of the w^{mtD1} inversion, having in the homozygous females a lethal

deficiency for part of the inert region including the *bb* gene, was produced. The structure of this inversion is shown in Fig. 1. When males bearing this inversion and an extra *Y*-chromosome (and therefore, showing red eyes instead of the usual light spotted eyes of flies without an extra *Y*-chromosome) are crossed with females with attached *X*-chromosomes, 99% of the sons, as well as the fathers, have an extra *Y*-chromosome. It is obvious that the irradiation of mature sperm of this line makes it possible to irradiate the sperms of the *XY* structure. The frequency of occurrence of deletions in the *XO* and *XY* sperms was compared when irradiated males were crossed with $\widehat{sc\ w}$ females. Ciphers 1, 2, 3, 4 in Fig. 1 indicate the breaks which give three types of deletions manifested in F_1 as sc^+w^{mt} (breaks 2 and 3), $sc\ w^{mt}$ (breaks 1 and 3) and sc^+w (breaks 2 and 4). It appeared that the deletions of the $sc\ w^{mt}$ class which seldom arise in the *XO* sperms (3 deletions $sc\ w^{mt}$ per 27 sc^+w^{mt} deletions) arise in great numbers (63 $sc\ w^{mt}$ deletions per 35 sc^+w^{mt} deletions, see the table) in the *XY* sperms; the other type of deletions arises with about equal frequencies in both series.

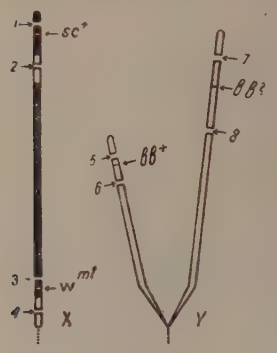

Fig. 1.

Dosage, r	Experiment	Sperm	♀♀ $\widehat{sc\ w}$	♀♀ sc^+w^{mt}		♀♀ $sc\ w^{mt}$		♀♀ sc^+w	
					%		%		%
4000	A I	*XY*	2673	13	0.49	20	0.74	1*	0.01
4000	B II	*XY*	1963	5	0.25	11	0.56	5	0.25
6000	A III	*XY*	1300	9	0.69	14	1.1	3	0.23
4000	B IV	*XY*	3606	8	0.22	18	0.49	5	0.14
4000	A I	*XO*	2423	15	0.62	2	0.08	8	0.33
4000	B II	*XO*	2014	5	0.25	0	0	7	0.35
4000	B IV	*XO*	1460	7	0.48	1	0.07	5	0.34

It was assumed that the $sc\ w^{mt}$ deletions are practically incomplete translocations between the *X* (break 3) and the *Y*-chromosomes; in fact, the acentric fragment of one of the *Y* arms, united with the centric fragment of the *X*-chromosome, gives the $sc\ w^{mt}$ deletion, while the additional fragments will be lost owing to non-union, and therefore the zygote, which ought to have developed into a super-female, due to non-union of the fragments will only produce a small duplication including the w^{mt} gene. During the formation of usual deletions, after irradiation the initial female bearing a deletion in F_1 will be devoid of the *Y*-chromosome, which must however be present when deletions are formed in the *XY* sperm. If the $sc\ w^{mt}$ F_1 females actually show a translocation of the expected type, they may bear only part of the *Y*-chromosome which, when these females are crossed with bb^{df} females, may either cover the lethal bb^{df} effect in the males of this cross, if the *bb* allelomorph is present in the *Y* fragment transferred into *X*, or not cover it, in which case no males will appear. However, the males appearing in the first case must be sterile, since they cannot bear the whole complex of fertility genes ([8]). Among the 31 $sc\ w^{mt}$ females crossed with bb^{df} males, 11 gave no males and 6 gave sterile males; 3 females were semisterile, 9 were sterile. In two cases when the $sc\ w^{mt}$ females were crossed with $w^{mt_4}Lw^{mt_{D1}}R$

* Counted by the end of the experiment. Experiments A and B were carried out in different apparatus.

males with an extra Y-chromosome, fertile males were obtained, as was to be expected, since the Y-chromosome absent in the female was obtained from the fathers' XY sperm.

Cytological investigation of the F_2 females *sc* w^{mt} showed in 5 cases a small extra rod-shaped chromosome consisting of a centric fragment of the X-chromosome located to the right of break 3, and an acentric fragment of the Y-chromosome, while in three cases this ought to be the distal Y^s fragment including, according to Neuhaus (^{3}), bb^+ (Fig. 2, a_1, a_2). In one case (Fig. *b*) an extra Y-chromosome with a shortened arm was found, this being probably caused by complex rearrangement. These cases point to the presence of the translocation in the Y-chromosome. According to the fragmentation hypothesis, the elimination of additional fragments must be the result of their non-union. However, if the acentric fragment of the active part

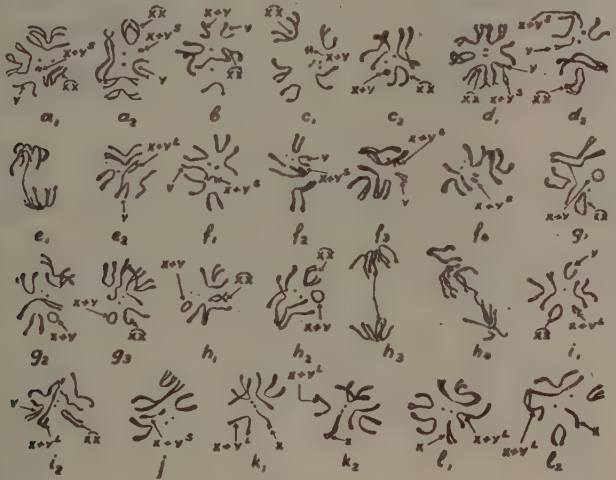

Fig. 2.

of the X-chromosome was to be eliminated as the result of the absence of the spindle fiber, the inert centric fragment of the Y-chromosome should be eliminated as the result of other causes. A cytological investigation of the chromosome in the oogonia of *sc* w^{mt} females in F_1 was undertaken with the purpose of finding the centric fragment and of studying the process of its elimination (whose presence would once more prove the correctness of the described structure of the *sc* w^{mt} females and, since it is not difficult to see, would refute the possibility of explaining the data obtained from the point of view of the described rearrangements at the diploid stage). Among the 43 *sc* w^{mt} females examined unsatisfactory mitoses were found in 18 females; in 11 females, besides a small extra rod-shaped chromosome which is a translocation of the acentric fragment of the Y-chromosome into the centric fragment of the X-chromosome, no centric fragments of the Y-chromosome with a «simple break» (Figs. c_1, c_2) were found; however, in 8 cases this fragment was found (Figs. *d*—*f*), while in 6 cases a half-ring or pseudoring in the shape of an almost closed horse hoof was found in addition to the normal set (Figs. *g*, *h*). Therefore, the centric fragment of the Y-chromosome which was not found during the genetic analysis in the progeny of the F_1 females, remains for some time in the germ duct of the initial F_1 females. In one case (Fig. *f*) when many mitoses were present, along with plates bearing the centric fragment of the Y-chromosome (8 mitoses in Figs. f_1,

f_2), one plate was found, where the short arm of the Y-chromosome with a break was absent (f_3) and one plate was devoid of the centric fragment (f_4). In other cases, as far as it is possible to judge on the basis of the few mitoses available, in the same female was found a variation in the size of a broken arm of the Y-chromosome (Figs. d_1, d_2). It is therefore obvious that a process of gradual elimination of the centric fragment takes place. This elimination is caused either by the destruction of chromosomes with a simple break, or, and this is more probable, especially in those cases when a centric fragment is found, by the lateral union of the chromatids of a centric fragment; owing to this, a closed system is formed anew which, however, owing to decentricity, leads to the formation of bridges and to a destruction of the centric fragment. If the lateral union of the chromatids with the unclosed surface of the break is a condition excluding, as the result of this mechanism, the more or less prolonged existence of simple breaks, the formation of permanently breaking bridges accounts for the destruction of fragments with «simple breaks». No sufficiently convincing pictures of the bridges were obtained with the help of aceto-carmin technique, however, some pictures indicate their possible formation (Fig. e_1). The lateral union of the chromatids apparently also causes formation of characteristic rings and of pseudo-hoof-shaped rings (Figs. g, h). Should a union of both centric fragments of the Y and X-chromosomes take place (break 3 in the X and random breaks in the Y), the newly formed dicentric chromosome, broken during mitoses, may form an almost inert chromosome with laterally closed chromatids, this chromosome containing the w^{mt} gene. Usually the above-mentioned X and Y translocations divide at mitoses before the other chromosomes (Figs. c_1, c_2, f_1, f_4). If this chromosome is sufficiently long, the lengthwise splitting chromatids will unite at the distal end and will be retained at the proximal end at the attachment point of the spindle fiber, thus forming a ring, or, if disjunction of the centromeres begins, an open ring. In this case bridges may also be observed (Figs. h_3, h_4).

Part of the deletions of the sc^+w type, obtained after irradiation of the XY sperm, may also be translocations between the X and Y-chromosomes. These translocations will arise at the breakage point 2 in the X-chromosome, at any break in the Y and at the union of the acentric fragment of X with the centric fragment of Y and at the non-union of additional fragments. Genetic analysis, similar to that described for the deletions of the $sc\ w^{mt}$ type, has shown that among the 8 fertile flies the expected translocation occurred in 3, while in one fly a deletion of the usual type was found. In one case the sterile F_1 males manifested the recessive bb gene, this pointing either to the presence of a translocation with the long arm of Y ([8]), or else to the position effect of the bb^+ in the short arm. The cytological analysis of two F_2 cases showed the presence of an extra rod-shaped chromosome, being a short arm of the Y-chromosome, while the long arm was substituted by the distal end of X which is invisible owing to its minute size (Figs. i_1, i_2). In one examined F_1 fly a translocation with the short arm of Y had taken place (Fig. j).

A similar genetic analysis of sc^+w^{mt} females obtained from sperms with an extra Y-chromosome, showed that in two sufficiently fertile cases these females had an Y-chromosome and a usual deletion, which was to be expected, since the majority of sc^+w^{mt} females ought to contain deletions of usual origin.

An additional class of reciprocal translocations between the X and Y-chromosomes was obtained. It is obvious, that in the case of break 3 in the X-chromosome and of any break in the Y-chromosome, and of the union of the acentric fragment X with the centric fragment Y (and the non-union

of two others), a deficiency of the w—N region will take place. Among a number of such deficiencies [some of them giving the lethal deficiency wN, and other viable w and w *rst* deficiencies (9)] six were analyzed cytologically and all showed the expected structure of the $X+Y$ chromosome (Figs. *k*, *l*). The frequency of occurrence of this rearrangement type is about the same as that of a corresponding additional class, i. e. as the frequency of occurrence of *sc* w^{mt} females.

Thus, it is clear that in the case of two indispensable breaks two fragments may unite in any order, while the other two remain unjoined and become eliminated. This confirms the fragmentation mechanism of the formation of chromosome rearrangements, and serves as a starting point for the explanation of X-ray induced sterility, and for the construction of other theoretical curves for the relation existing between rearrangement frequency and dosage of X-rays, and for other problems of the theory of the mechanism of the rearrangement process.

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GENETICS

FREQUENCY OF OCCURRENCE OF ROENTGENOMORPHOSES AS DEPENDENT UPON X-RAY DOSAGE

By V. B. SOLODOVNIKOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 3. IX. 1940)

The phenomenon of parallelism of hereditary and non-hereditary variation has received application in (1) the study of phenogenetic problems and in (2) the comparative study of the mechanism of action of external factors on the somatic and germ cells.

My previous work ([3]) on the after-effect of X-rays was carried out in the last direction. Wishing to continue my investigations in the same direction, I now decided to examine the relation existing between the frequency of occurrence of roentgenomorphoses and the dosage of X-rays. Friesen ([2]) showed that there is a direct dependence between the dosage applied and the general frequency of roentgenomorphoses. Besides, Friesen observed a certain specificity in the occurrence of morphoses after application of different X-ray dosages in *Drosophila simulans* and *D. hydei*.

However, since Friesen used two dosages only and did not apply the statistical method of recording the morphoses, one can hardly speak of a direct, or any other dependence between the X-ray dosage and the frequency of occurrence of morphoses on the basis of his experiments. Yet to clear the character of this relation is of great interest for the comparative study of the mechanism of the action of X-rays on the variation of the germ cells and on the morphogenetic reactions.

The experiments were carried out on a wild line of *D. melanogaster* termed «Samara». Larvae of only a definite age were irradiated, since it is known that the appearance of phenocopies is confined to a definite sensitive period. Uniformity in age is attained by transitory mass ovipositions. The following conditions of treatment were chosen: KV—50, mA—4, filter—Al 1 mm. Variation in dosage was attained by changing the duration of exposure. Four dosages were applied: t_1=17′30″ (1000 r), t_2=35′ (2000 r), t_3 = 53′30″ (3000 r) and t_4 = 71′ (4900 r).

In order to obtain more objective evidence on the relation between the frequencies of morphoses and X-ray dosages, I studied one type of morphoses only. It was assumed that the regularity thus found will be valid for the occurrence of other types of morphoses as well. A morphose, frequently occurring under these conditions, was taken, namely the KN effect of Goldschmidt ([1]). This morphose is sufficiently well studied. Besides, the effect of irradiation can in this case be estimated both after the per cent of flies changed and the relation between the expression of this character (in the individuals changed) and

the X-ray dosage. The treatment was applied at the stage sensitive to the character under consideration (at the age of 96—110 hours).

The results of the experiments are summarized in Tables 1 and 2. Experiment 1 (Table 1) clearly shows that an increase in the X-ray dosage causes an increase of the per cent of morphoses, this increase bearing the character of a regular direct dependence. A comparison of the frequencies calculated with the values observed shows that there is scarcely any difference ($x^2=1.0589$, $Pg=0.78$).

Table 1

No. of experiment and X-ray dosage (in min)	Results of counts of morphoses: Total number of examined flies	Number of morphoses found	Morphoses %	$m\%$	Diff / m_{diff}	Diff / m_{diff}	Expected frequencies calculated according to Markov, proceeding from direct dependence	Age of larvae by the beginning of irradiation (in hours)
1	2	3	4	5	6	7	8	9
No. 1 Control (not X-rayed) .	991	11	1.1	±0.39				
t_1 —22 . . .	496	44	8.82	±1.33	7.72±1.36	5.6	8.026	96—105
t_2—35 . . .	228	27	11.61	±2.12	2.79±2.5	1.1	13.77	
t_3—54.5 . .	581	104	17.9	±1.6	6.29±2.91	—	21.643	
t_4—71 . . .	847	253	29.8	±1.57	11. 9±2.02	—	28.766	
No. 2, t_2—35 .	617	34	5.51	±0.92	6.93±1.76	3.9	5.55	112—123
t_4—71 .	418	52	12.44	±1.51			12.26	
No. 3, t_1—17.5 .	175	14	8	±2.45	11.42±3.62	3.15	8.36	96—105
t_2—35 .	242	47	19.42	±2.54	0.45±4.17	0.10	16.22	
t_3—53.5 .	148	28	18.97	±3.31	16.27±5.55	2.93	24.48	
t_4—71 .	105	37	35.24	±4.46			32.49	
No. 4, 3000 r, continuous X-raying . .	2547	601	23.6	±0.26				
3000 r, interrupted X-raying (2 intervals by 5 min.) . .	1788	509	28.52	±1.06	4.82±1.21	3.98		110—120
Total of No. 1 and No. 2 t_1 . .	671	58	8.67	±1.08	7.07±1.99	3.58	8.07	96—105
t_2 . .	470	74	15.74	±1.66	2.27±2.17	1.04	14.858	
t_3 . .	729	132	18.11	±1.39	12.36±2.06	6.00	21.644	
t_4 . .	952	290	30.47	±1.52			28.431	

In experiment 2 two dosages were applied (2000 r and 4000 r). Larvae at the upper limit of their sensitive period were irradiated; some of the larvae had already passed the sensitive stage, which affected the absolute values of the frequency of morphoses in the experiment (column 5). A comparison of the per cent of morphoses ($t_2=5.51$, $t_4=12.44$) shows that there is a direct relation between the dosage and the frequency of roentgenomorphoses, as well as a direct proportionality between them, since a double increase in the dosage makes the per cent of roentgenomorphoses rise double, too. Therefore, the first experiment had to be repeated (experiment 3). Despite the small amount of material available in experiment 3, it becomes clear that the general picture of the frequency of occurrence of roentgenomorphoses approaches that of a direct relation existing between them and the X-ray

dosage. The dosage of 3000 r presents an exception, causing a departure from this direct dependence. This may, possibly, be explained by the fact that perfectly identical stages cannot be obtained in every experiment, since the age of the larvae greatly depends on population density in the culture, on humidity, on chemical reaction of the environment, etc.

As experiments 1 and 3 were carried out under almost identical conditions (age of the larvae during irradiation, irradiation dosages, etc.) and the results were rather similar, it seems advisable to sum up the data of these two experiments and to evaluate them statistically. The direct relation existing between the occurrence of morphoses (and in particular its direct proportionality) and the X-ray dosage becomes still more conspicuous when the data are summed up (compare values for t_1 and t_2, t_2 and t_4).

A special examination of the fly-morphoses showed that the manifestation of the morphose caused by higher X-ray dosages is more clearly expressed than that due to the action of lower dosages. For instance, the application of the small t_1 dosage causes small notches in the distal part of the wing, while t_2 causes morphoses with deep wing notches. Owing to this, the wing becomes almost twice as short and flies with wings having the aspect of antlers are met with. In order to evaluate objectively this phenomenon, I applied the scale worked out by Goldschmidt for the KN effect.

Table 2 shows that there is a direct relation between the size of the notch and the dosage of X-rays. The increase in the X-ray dosage is followed by an increase in the manifestation of the notch. Thus, the existence of a direct relation between the expression of the morphoses and the dosage of X-rays is quite obvious here. No conclusions as to the direct proportionality can be made in this case. First, owing to scarcity of material, and secondly, because Goldschmidt's scale shows a number of defects.

Table 2

Relation between Expression of Morphoses and X-ray Dosage

X-ray dosage	Distribution of roentgenomorphoses according to the classes of Goldschmidt's scale (in %)									
	I	II	III	IV	V	VI	VII	VIII	IX	
	Nicked (Mohr)	Notched (Mohr)	Notched (Mohr)	Scalloped (Morgan school)	Scalloped (Morgan school)	Snipped (Muller)	Ragged (Mohr)	Antlered (Morgan school)	Strap (Morgan school)	General %
t_1	3.62	3.42	1.00	0.62	0.22	—	—	—	—	8.82
t_2	2.64	3.96	2.20	1.32	0.88	0.44	0.44	—	—	11.61
t_3	3.10	4.65	4.48	2.23	1.72	1.22	0.31	0.18	—	17.9
t_4	3.07	6.73	6.73	7.91	2.24	1.29	0.47	0.46	0.59	29.8

Besides the study of dependence of the frequency of roentgenomorphoses and size of the notch on the dosage of X-rays, a special experiment (No. 4) was carried out (Table 1), in order to find out the influence of the factor of time upon the rate of occurrence of roentgenomorphoses. Some investigators have shown that duration of treatment does in no way affect the rate of the mutation process. However, experiment No. 4 shows that time plays some part in the occurrence of roentgenomorphoses.

The «direct proportionality» existing between the X-ray dosage and the frequency of occurrence of changes in the cell will be briefly discussed here on the basis of the evidence obtained by me and a number of other students. The existence of a «direct proportionality» has been proved with mathematical accuracy for the case of occurrence of mutations. As a rule, the interpretation of this phenomenon is reduced to that of the changes of the gene as a monomolecular reaction. Lobashov (5) has shown that the «direct proportionality» found in the experiments on mutations may be understood as a cumulative effect of different mutation frequencies preserved as a result of processes of germinal selection proceeding with a different intensity (according to the dosage applied) at various maturation stages of the germ cells. It seems to me that such an interpretation clears but one side of this complicated phenomenon.

Neither the origination nor the manifestation of morphoses is associated with germinal selection, in spite of the existence of a «direct proportionality». However, the direct relation between the expression of the morphose (size of wing notch) and the dosage of X-rays is apparently due to the different dynamics of cell elimination during the process of cytolysis. The «direct proportionality» between the number of arising changes and the X-ray dosage is apparently a more general regularity, characterizing the processes taking place in the changes of the traumatized cell. This is confirmed by a number of experiments on the relation between non-disjunction of *X*-chromosome and dosage of X-rays (6). These experiments go to confirm a direct dependence between the non-disjunction of the chromosomes and the dosage of X-rays, while a strict direct proportionality is found for some X-ray dosages, as well as for the occurrence of «point» mutations. A similar picture is observed in my experiments with morphoses and in the experiments of Blanck and Braun(7) on phenocopies. The conclusion may, therefore, be drawn that the existence of a «direct proportionality» should not be taken to indicate a specificity of the gene's changes under the effect of X-rays, at least from the quantitative point of view. It rather points to the specificity of the effect of the X-rays and that of the mechanism of reaction of the traumatized cell, than to the specificity of the structure of the gene affected by X-rays. In connexion with this interpretation of the action of X-rays on the appearance of mutations, the idea of an indirect action of X-rays on the change of the gene seems to be more probable, namely, through changes of the cell as a physiological system, i. e. changes similar to those taking place during the production of morphoses, non-disjunction of chromosomes and other changes in the cell.

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GENETICS

CHIMERAS IN BARLEY

By S. J. KRAJEVOJ

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We crossed in 1937 *Hordeum sativum* var. *nutans* Schübl. (No. 1012, catalogue of collections of the Institute of Plant Industry) with *Hordeum sativum* var. *compactum* Kraj. (1). *H. s.* var. *nutans* has a loose spike (Fig. 1, 40529), its glumes are linear-lanceolate, narrow (less than 1 mm wide). Awns are long, bearded, straw-yellow and exceed in length the spike from 1.5 to 2 times. *H. s.* var. *compactum* has compact spikes (Fig. 1, 40612), the glumes are linear-lanceolate, narrow (less than 1 mm in width). The awns are smooth and twice as long as the spike; both spikes and awns are light maroon.

The characters of *H. s.* var. *nutans* showed dominance in F_1; in F_2 there occurred Mendelian segregation. In 1940 in a heterozygous family F_3 40549/20, a single plant was found which had developed 11 culms, 10 of which bore spikes of the *nutans* type (Fig. 2, 40549 N), while the eleventh culm bore a spike which was quite distinct from the rest and approached morphologically the *compactum* type (Fig. 2, 40549 ch). Though we have been working with barley throughout these last four years, we never happened to see chimeras before. Aside from the case described above, we first happened to observe chimeras in 1940 in a hybrid material showing variation in leaf and stem colour. The first chimera had arisen in F_3 from a cross *H. vulgare* var. *pallidum* Sér. (from Algeria, No. 4647, collection catalogue of the Institute of Plant Industry) × *H. vulgare* var. Dundar-beyi Zhuk. (from Japan, No. 9223, collection catalogue of the Institute of Plant Industry). Two culms (out of 9 culms developed by a single plant) were chimeral. Their chimeral nature was manifest in white strips on the leaves, the variegated leaves being distributed on one side of the stem, those on the other side being of normal coloration. The stem was stripped, too, on the side which bore stripped leaves. The strips passed from the stem to the spike, so that half of the latter was also variegated: some of the paleae were entirely devoid of chlorophyll, some bore white strips. All the flowers were fertile and produced grains. The second culm was similar to that described above and bore a chimeral spike of the same kind. Whether or not the character found in the chimera is hereditary, will be checked the next year. The remaining culms were normal.

Chimeras of the same type were observed in two following crosses by V. F. Savinskaya (unpublished data). Cross No. 1: *Hordeum vulgare* var. *pallidum* Sér. (from Morocco, No. 7570, collection catalogue of the Institute of Plant Industry) was crossed with *H. v. parallelum* Körn. (from Japan,

No. 1169, collection catalogue of the Institute of Plant Industry); the former was the pistil parent, the latter—the pollen parent. In the family 445/30 a single plant segregated out, showing partly stripped leaves; it happened to be of the winter type and produced no offspring (spring sowing). Cross No. 2: ♀*H. vulgare* var. *pallidum* Sér. (from India, No. 10581, collection catalogue of the Institute of Plant Industry)× ♂*H. v.* var. *brevispicatum* Vav. et Orl. (from Japan, No. 10988, collection catalogue of the Institute of Plant Industry). In the F_3 family 448/4 there was one plant of the winter type which had part of its leaves white stripped, the white sectors covering up to a half of the leaf's width.

In wheat chimeras are of more frequent occurrence. Thus, Åkermann (4), working with the cross Ironwheat type × Thulewheat type, has found in F_2

Fig. 1.

Fig. 2.

one plant which had a spike of chimeral structure; it bore paleae of two sorts. On the one side of the spike there were paleae of the speltoid type, while on the other the paleae were of the normal, *vulgare* type. However, kernels collected separately from the two morphologically distinct parts of the spike, produced entirely normal plants of the *vulgare* type. Among the offspring of the sibs of the chimeral plant there were no plants with speltoid spikes. After having discussed several hypotheses as to the origin of the chimera in question, the author arrived at a conclusion that the progeny test has proved that changes of speltoid character have affected only the epidermal layer; otherwise, in case the subepidermal tissue giving rise to spores was affected, the result would be a different one. In an other case Åkermann (4) has found a speltoid chimera of the same type in the F_5 of a cross between two forms of the *vulgare* type. Progeny test has shown that this chimera, too, produced only normal progeny. In this case he was probably dealing with periclinal chimeras, in which the normal *vulgare* core was surrounded with a heterozygous epidermis. In another case (4) a variegated speltoid chimera was found which produced in the progeny heterozygous speltoids and normal *vulgare* plants in the 5 : 1 ratio. It follows from this that this was a sectorial chimera possessing more than two external cell layers of the *vulgare* type.

The author thinks that the chimeras observed by him were due to mutation in the cells of the growing cone, involving a change in one of the two factors, necessary for the development of the normal *vulgare* type.

There occur chimeras in wheat hybrids with regard to colour ([2]); progeny test has similarly shown that only the external integuments of the spike were affected. Lepin ([2]) has discovered in the F_3 a chimera with regard to pubescence of the spike. The hybrid, in which the chimera in question had arisen, was obtained from a cross pubescent wheat×non-pubescent wheat. Progeny test has shown that only outer integumental parts of the spike were affected in this chimera; the spike itself was heterozygous for pubescence. this condition resulting in unifactorial segregation. The author assumed that the heterozygous plant had probably suffered vegetative segregation in the growing cone, owing to which the chimeral spike had arisen.

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PFLANZENPHYSIOLOGIE

QUARZ ALS NEBENFAKTOR BEI ERFORSCHUNG DER MITOGENETISCHEN STRAHLUNG

Von M. MOISSEJEWA

(Vorgelegt von A. A. Richter, Mitglied der Akademie, den 22. X. 1940)

Zum Beweis mitogenetischer Strahlung wurden im Laufe von 15 Jahren seit ihrer Entdeckung eine Anzahl sowohl biologischer als physikalischer Methoden dargeboten. Viele dieser Methoden wurden einer scharfen Kritik unterworfen und für untaugliche erklärt (1). In den letzten Jahren gibt die Schule von Gurwitsch «die chemische» oder «kolorimetrische» Methode von Potozky mit Scheiben von roter Rübe und Blütenblättern als besonders genaue Methode (2) an. In Hinsicht darauf. dass auch diese Methode seitens des deutschen Physikers Gerlach (3) eine vernichtende Kritik erlitt, scheint es zweckmässig die Genauigkeit dieser Methode auf experimentellem Wege zu prüfen. Auf Grund einer Reihe von Versuchen konnten folgende Schlüsse gezogen werden (4):

1. Die Methode von Potozky mit Scheiben von roter Rübe und Blütenblättern ist keine chemische, für die sie die Schule von Gurwitsch hält, sondern eine biologische, weil hier lebende Zellen des Gewebes der Pflanze als Detektor dienen.

2. Bei Wirkung der chemischen Reaktionen $K_2Cr_2O_7 + H_2SO_4$ und $KMnO_4 + H_2O_2$ auf Scheiben von roter Rübe, die mit Wasser an den Quarzboden (in der Kontrolle Glasboden) des Zylinders, in dem die Reaktion unter Bedingungen unhermetischer Isolation und ungenügender Befeuchtung vor sich geht, geheftet sind, wird der Effekt von Potozky beobachtet, d. h. man bemerkt eine Beschleunigung der Rötung des Wassers und eine Verstärkung der Intensität der Färbung in der Induktion.

3. Bei Versenkung des Detektors in Wasser, unter Bedingungen seiner hermetischen Isolation und bei auf 2 mm entfernter Induktion wird ein solcher Effekt nicht beobachtet.

4. Bei Wirkung der chemischen Reaktion $KMnO_4 + H_2O_2$ auf Blütenblätter zarter Blumen (hellrosa und weisse Gartennelken, weisse und hellila Levkojen, Schneeglöckchen) durch den Quarzboden in einer Entfernung von 1 mm unter Bedingungen unhermetischer Isolation des Detektors und ungenügender Befeuchtung, wurde der Effekt von Potozky in vielen Fällen beobachtet, d. h. die Blütenblätter, die sich unter Quarz befinden, werden nach 6—7 stündiger Dauer des Versuches durchsichtig und welken früher, als diejenigen, die unter Glas waren.

5. Bei Versenkung der Blütenblätter in Wasser und ihrer hermetischen Isolation bei der Induktion in Entfernung von 1—2 mm, unter Anwendung vollkommen gleichartigen Materials (Blütenblätter von ein und derselben Stelle, ein und derselben Blume, Innen- oder Aussenblätter ein und derselben Grösse usw.) wird überhaupt kein Effekt beobachtet.

6. Wenn man auf ein heizbares Objekttischchen zwei Platten (eine aus Quarz, die andere aus Glas) anbringt, eine jede auf 2 Glaskapillaren von 1 mm Dicke gestützt, und auf diese Platten unter Deckglas in Wassertropfen Blütenblätter im § 4 erwähnter Blumen legt, so wird bei Erwärmung von 45—50° C im Laufe von 15—20 Minuten der Effekt von Potozky hervorgerufen. Mit Scheiben von roter Rübe kann der Effekt von Potozky unter denselben Bedingungen auch bei niederer Temperatur (bei 35—40° C) eintreten. Wenn gut angefeuchtete Blütenblättchen auf dem heizbaren Objekttischchen ohne Deckglas liegen, so tritt der Effekt von Potozky bei niederer Temperatur gleichfalls auf.

7. Die grössere Wärmeleitung des Quarzes im Vergleich mit Glas lässt sich leicht mit Hilfe des Paraffins beweisen. Wenn man ganz gleiche Stückchen Paraffin auf Platten von Quarz und Glas legt, welche auf dem bis zum Schmelzpunkt des Paraffins erhitzten Objekttischchen in der Weise, wie in § 6 angegeben wurde, angebracht sind, so bleibt bei Loslösung dieser Stückchen nach 20 Minuten auf der Quarzplatte eine dicke Schicht geschmolzenen Paraffins, auf Glas dagegen ein kaum bemerkbarer Abdruck.

8. Auf Grund §§ 6—7 kann man schliessen, dass Dank der grösseren Wärmeleitung des Quarzes [10 bis 20 Mal grösser als die des Glases ([4])] die Scheiben der roten Rübe und die zarten Blütenblätter, welche im § 4 angegeben wurden, auf Quarz erheblich mehr Wärme erhalten, als auf Glas, wodurch der obenerwähnte Effekt entsteht.

9. Die Blütenblätter roter Nelken, Astern aller Farben, gelber und roter Georginen zeigen sogar bei Erwärmung bis 56° bei 25—30 Minuten Zeitdauer des Versuches gar keinen Unterschied, nicht nur auf Quarz und Glas, sondern auch beim Vergleich mit nicht erwärmten Blütenblättern.

10. Auf Grund der Kontrollversuche auf Quarz und Glas mit Blütenblättern, die in § 4 angegeben wurden, kann man schliessen, dass das von Potozky «möglichst frisch» ([3]) benutzte Versuchsmaterial nicht gleichartig war, denn die Verschiedenheit der Reaktion auf Temperaturerhöhung hängt von Alter und Frische der Blume ab. Ein inneres Blütenblättchen, welches einer eben erschlossenen, oder auch eben gepflückten Blume entnommen wurde, reagiert später auf die Temperaturerhöhung im Vergleich mit äusseren Blütenblättchen, am vorhergehenden Tage erblühter, oder gepflückter Blume.

11. Auf Grund des dargelegten darf es für bewiesen gelten, dass der Effekt von Potozky mit Scheiben von roter Rübe und mit Blütenblättern nicht von mitogenetischer Strahlung bedingt wird, sondern in Folge der Einwirkung äusserer Faktore, nämlich eine Begleiterscheinung bei Aufstellung der Versuche, entsteht.

12. Als einen solcher Faktoren, im Zusammenhang mit ungleicher Wärmeleitung bei Quarz und Glas, muss die geringe Temperaturerhöhung anerkannt werden, welche die synthetischen Reaktionen begleitet und wohl die Wasserverdünstung, sowohl als das Vertrocknen zarter Gewebe beschleunigt und damit vielleicht auch die Permeabilität des Protoplasmas ändert.

13. Es kann die Meinung geäussert werden, dass die Analyse der biologischen Methoden ihre Rolle schon ausgespielt hat. Die Tatsache der Untauglichkeit der Wurzel- ([1]), Hefenmethode ([5]), der sogenannten «kolorimetrischen» Methode von Potozky ([6]) und anderer, gibt genügende Ursache zu vollkommen begründetem Zweifel an der Existenz mitogenetischer Strahlung.

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PLANT PHYSIOLOGY

CREOLIN AS A POWERFUL FUNGICIDE

By V. P. YAGODKINA

(Communicated by A. A. Richter, Member of the Academy, 14. IX. 1940)

At the Plant Protection Department of this Institute one was faced with the task of trying the effect of various fungicides in controlling the *Botrytis cinerea* and some other moulds, injurious to oil crops in the period of vegetation as well as germination and harvesting. Not only those generally used, but also some rather rare fungicides were put to the test. Their effect was studied on pure cultures of fungi.

For that purpose the surface of the nutrient agar which was to be seeded upon with spores, was coated with a thin layer of the respective poison (either a fluid or dust), whereupon followed inoculation. On to the poisoned surface the spores were not transferred by means of a needle, but were simply poured out from a dish in order to prevent them from getting inside the agar and out of the contact with the poison. The needle or forceps were used, however, in the tests with developed mycellium instead of spores. A bundle of mycellium fibres taken in that way from the dish with the pure culture was shaken down on the poisoned surface of the agar.

The substances tested for their fungicidal power against *Botrytis cinerea* were these: 1) Bordeaux mixture (1% solution); 2) blue vitriol (1% solution); 3) flowers of sulphur and ground sulphur (dry); 4) sulphur and lime decoction (28° Beaumé, no dilution); 5) granosan (dry); 6) kerosene (soap and kerosene emulsion); 7) lime (milk and air slaked); 8) ashes (from sunflower, dry); 9) creolin.

Creolin was used as a 5% emulsion (poured or sprinkled over agar surface) or as vapour. In the latter case a small piece of cotton-wool (1 mm^3) soaked with creolin was fastened to the inside of the lid covering the Petri dish which contained a perfectly pure agar, inoculated with the spores of *Botrytis cinerea* or some other of the moulds tested (as *Penicillium*, *Aspergillus*, *Fusarium*, *Rhizopus* etc.). In further tests the cotton-wool was replaced by merely a cross-like smear of creoline. It is in that way that the spores were let to germinate in the presence of creolin vapour.

Their germination in every dish was followed up from the next day after inoculation onwards. The observations were carried on with a microscope. In Table 1 are brought together the results.

The spores and, as further tests have shown, even the developed mycellium of *Botrytis cinerea* are fatally affected by granosan and creolin. That they completely lose all vital power is borne out by their refusal to germinate in another dish on freshly poured agar in the absence of creolin.

Table 1

Influence of Certain Substances on Germination of Spores and Development of Mycellium of *Botrytis cinerea*

Substance and method of use	Spore germination and development of mycellium after inoculation			
	2nd day	3rd day	4th day	6th day
Bordeaux mixture (1%) poured over agar	×	×	××	×××
Blue vitriol (1% water solution), poured over agar	×	×	××	×××
Flowers of sulphur and ground sulphur, sprinkled as dust	×	××	×××	×××
Sulphur and lime decoction, poured over agar	+	×	×××	spores
Granosan sprinkled as dust	+	+	+	+
Soap and kerosene emulsion, pouring	×	××	×××	×××
Air slaked lime, sprinkling	0	0	+	+
Lime white, pouring	+	+	+	+
Sunflower ashes, sprinkling	0	0	+	+
Creolin emulsion, pouring	0	0	0	0
Creolin vapour	0	0	0	0
Control, unpoisoned agar	×	××	×××	spores

Notation: 0—no germination; +—very weak germination of a few spores; × — strong germination all over the surface (seedling length includes several spore diameters); ×× — appearance of microscopically observable air mycellium; ××× —formation of normal fungous down, visible by unaided eye; spores—spore formation.

Fungicidal qualities of the creolin vapour can be turned to account in controlling the *Botrytis cinerea* during drying of castor-oil plants and at the time of harvesting peanuts. Experience shows that both the castor-oil plant and peanuts (as well as several other crops) are often attacked by *Botrytis cinerea* during harvest, which is naturally followed by a decrease in yield quality.

What would seem to be the most simple way of treating the harvested crop for affection with *Botrytis cinerea* is to spray it with creolin emulsion. But this is bound to increase the humidity of the material at the time when it has to be subjected to drying, and therefore is not only undesirable but can even be injurious in the cool and short autumn days.

In order that moisture should not be introduced in this way, we made an attempt to prepare creolin dust. As ingredients we tried: 1) talc which with as much creolin in it as 20% of its own weight still remained a fine powder; 2) road-dust which could absorb 12% of creolin on the average; 3) air slaked lime, able to absorb 16% of creolin.

Lime proved to be the most suitable ingredient on account of its alkaline reaction, which inhibited the development of *Botrytis cinerea* as well as of other fungi. The effect of dusts prepared in the above manner was first tested on moulds: *Penicillium*, *Aspergillus*, *Alternaria*, *Fusarium*, etc., all attacking the seed of oil crops and reducing its germinating power. *Botrytis cinerea* was not at our disposal by that time.

Among several different preparations the creolin dust was tried as seed disinfector from moulds. The other substances tested were the AB preparation, blue vitriol (anhydrous and in solution), Paris green, granosan and a number of new ones prepared at the Institute of Fungicides.

Among the treated seeds were those from peanuts, castor-oil plant and soya. Germination of control seeds took place in Koch dishes on filter paper. The AB preparation and Paris green were excluded after the first tests, for

they had too strongly reduced the germinating power of the seeds, both at the laboratory and under field conditions.

Blue vitriol in solution was found suitable only in the case of castor-oil seeds. With peanut and soya wet treatment was not successful, because these seeds had become quickly swollen, which had made sowing difficult and also affected the germination power unfavourably. As for the use of non-aqueous commercial blue vitriol as powder, great care must be exercised, for, as we have observed it in our tests, the seeds are apt to get scorched under its action. The results of the experiment are shown in Table 2.

Table 2

Treatment of Peanut Seeds with Various Disinfectors. Its Influence on Germination and Development of Moulds under Laboratory Conditions, 1939

Disinfectors	Germination, %	Seeds affected with moulds, %
Creolin dust	96	0
Granosan	96	0.5
AB preparation	73	29
Blue vitriol (non aqueous)	55	14
Paris green	0	61.5
Control	94	64

As is seen from Table 2, the first place with regard to disinfecting power is held by the creolin dust. In some of our preliminary tests promising results were also obtained with it in treating wheat seeds for smut; germination of wheat under these conditions was also successful.

The effectiveness of creolin dust as a chemical measure for controlling *Botrytis cinerea* was tested under industrial conditions on peanut shrubs, while they were being laid in store in layers. Every layer was sprinkled with creolin dust. Later on, when the beans were plucked, only isolated cases of slight affection with *Botrytis cinerea* were observed among the dusted stocks, while the undusted were affected up to 100%. Owing to strong affection, the leaves of undusted plants were completely rotted. This loss of leaves during after-ripening affected the yield qualitatively, and what is more, the beans themselves suffered considerably from rotting.

Positive results were also obtained in dusting castor-oil bunches affected with *Botrytis cinerea*, after they had been moved from the field to the thrashing floor.

At present we are successfully using the creolin dust as a measure against *Botrytis cinerea* in pot experiments with live plants.

So we have every reason to conclude this article with saying that the creolin dust is a valuable means of controlling the *Botrytis cinerea* and other hyphomycetous fungi at the time of harvesting, in stacking late crops, in storing fruits and roots, as well as fibres. The biting smell of creolin is not a lasting one. The fungicide is cheap, harmless for man and animals. It should be videly tried on an industrial scale in the next season.

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PFLANZENANATOMIE

MÄNNLICHE GAMETEN BEI ANGIOSPERMEN

Von W. W. FINN

(Vorgelegt von A. A. Richter, Mitglied der Akademie, den 7. XII. 1940)

Im Anfang unseres Jahrhunderts war unter den Botanikern die Ansicht vorherrschend, dass die generative Zelle bei Angiospermen ihr Plasma im Entwicklungsprozess gewöhnlich verliert, infolgedessen sollten die männlichen Gameten Bildungen, die kein eigenes Plasma besitzen, darstellen, die sogenannten «Spermakerne».

Auch ich nahm dann auf Grund von Literaturangaben an, dass die männlichen Gameten im Evolutionsprozess einer stark ausgeprägten Reduktion unterworfen sind, deren Endresultat zu plasmalosen Spermien führt. In meiner mit dem Altmeister S. G. Nawaschin (1) gemeinsamen Arbeit ist dieser Frage ein ganzes Kapitel gewidmet. Meine weiteren Forschungen, sowie die Arbeiten meiner Schüler und vieler Botaniker stellten fest, dass die Spermien in Form ganzer Zellen bei Angiospermen nicht nur vorkommen, sondern sogar sehr stark verbreitet sind. Als Beweis dafür dient ein von mir zusammengestelltes Verzeichnis der Angiospermen, bei denen Spermazellen beobachtet wurden (2). Es enthält mehr als 230 Arten, die zu 27 Reihen und 64 Familien gehören, worunter sowohl Dicotyledonen, als auch Monocotyledonen zu treffen sind. Die männlichen Gameten in Form ganzer Zellen wurden nicht nur in den Pollenkörnern und Pollenschläuchen verschiedener Vertreter der Angiospermen beobachtet, sondern man konnte sie auch in den Embryosäcken einer Anzahl von Arten, die im System zu fernstehenden Vertretern dieser Pflanzen gehören, konstatieren.

Zu diesen Arten gehören folgende: *Vallisneria spiralis* [man konnte eine der Spermazellen zu wiederholten Malen in unmittelbarem Kontakt mit der Eizelle beobachten (3)], *Plumbagella micrantha* [Spermium in Form einer ganzen Zelle im Kontakt mit dem sekundären Kern des Embryosackes (4)], *Saxifraga granulata* (5), *Myricaria germanica* (6, 7), *Asclepias Cornuti* (8) und andere.

Nach der Veröffentlichung des obenerwähnten Verzeichnisses (2), setzte ich das Studium der Entwicklungsgeschichte der Spermien bei einer Anzahl von Angiospermen fort, die in dieser Hinsicht zu noch nicht untersuchten Familien gehörten. Dabei stellte ich männliche Gameten in Gestalt ganzer Zellen bei *Oxycoccus palustris*, *Calluna vulgaris* und *Euphorbia palustris* fest (nicht veröffentlichte Angaben); ausserdem wurde neuerdings in unserem Laboratorium die Existenz von Spermazellen bei *Geranium sanguineum* und *Tropaeolum majus* konstatiert, was die Zahl der Familien mit befruchtenden Elementen in Gestalt ganzer Zellen noch vergrössert (nicht veröffentlichte Angaben).

Auch in der Literatur erscheinen immer wieder neue und neue Angaben über das Vorhandensein von Spermazellen bei verschiedenen Vertretern der Angiospermen. So führt Schnarf (⁹) in einer unlängst erschienenen Arbeit, die den Variationen der Struktur des Pollenkorns bei Angiospermen gewidmet ist, noch 7 Arten an, die in meinem Verzeichnis nicht vorkommen, nämlich: *Juncus articulatus*, *Thrianthema monogyna*, *Potamogeton perfoliatus*, *Sambucus racemosa*, *Mentha Requienii*, *Dionaea muscipula* und *Petiveria alliacea*. Die zwei ersten Arten gehören zu den von mir nicht angegebenen Familien.

Trankowsky (¹⁰) berichtet, dass die männlichen Gameten sich auch bei *Lathyrus odoratus* als Spermazellen spindelartiger Form mit zugespitzten Enden erwiesen. Obgleich Eigsti (¹¹) darauf direkt nicht hinweist, dass die Gameten bei *Polygonatum commutatum* Zellen sind, allein auf seiner Figur 51 sind sie deutlich sichtbar.

Wie soll man dann die in der Literatur bis jetzt vorkommenden Hinweise auf das Vorhandensein plasmaloser männlicher Gameten bei einigen Angiospermen erklären?

Eine Nachuntersuchung, die in unserem Laboratorium vorgenommen worden war, zeigte bei einer Anzahl solcher Arten, dass diese Angaben auf die Unzulänglichkeit der angewendeten primitiven Methoden zurückzuführen seien. Mit Hilfe einer vollkommener Forschungsmethode gelang es uns bei solchen Arten die Existenz von Spermazellen festzustellen. Es muss auch noch darauf hingewiesen werden, dass sich der generative Kern bei einigen Angiospermen, wie auch die Spermakerne, stark ausreckt, so dass es sehr schwer ist bei gewöhnlichen Untersuchungsmethoden die eigene, dünne Plasmaschicht der männlichen Gameten zu finden. Einen solchen Fall haben wir z. B. in der Familie *Compositae* (¹²).

Eine besondere Bedeutung kommt bei der Entscheidung der uns interessierenden Frage den neuesten, an lebenden Objekten ausgeführten Forschungen (¹³⁻¹⁸) zu. Mit Hilfe dieser Methode ist schon eine bedeutende Anzahl der Vertreter der Monocotyledonen wie auch der Dicotyledonen untersucht worden, bei denen das Vorhandensein der Spermien in Gestalt ganzer Zellen nachgewiesen werden konnte.

Von besonderem Interesse ist die Tatsache, dass unter diesen Arten das klassische Objekt *Lilium martagon* figuriert, bei welchem, nach der Meinung vieler Botaniker, die Existenz plasmaloser Spermien endgültig bewiesen war.

Meine Mitarbeiter setzen jetzt die Untersuchungen an lebenden Objekten fort, welche immer wieder neue Resultate ergeben, die dem allseitigen Studium der Entwicklungsgeschichte und Struktur der männlichen Gameten der Angiospermen äusserst beitragen. Diese Arbeiten entscheiden definitiv im positiven Sinne die so lange bestrittene Frage nach der Existenz der Spermien in Gestalt ganzer Zellen bei allen Angiospermen.

Meiner Ansicht nach, kann diese Frage gegenwärtig als endgültig entschieden betrachtet werden, umsomehr da das faktische Material mit den Erwägungen a priori völlig übereinstimmt, worauf ich schon unlängst hingewiesen habe (¹⁹); folglich kann man Spermien in Gestalt ganzer Zellen als allgemeines Merkmal für alle Vertreter der Angiospermen annehmen.

In Anbetracht des obenerwähnten muss bei der Charakteristik des Pollens dieser Pflanzen der Terminus zweizelliger und dreizelliger anstatt zweikerniger und dreikerniger Pollen gebraucht werden.

Die Untersuchungen letzterer Zeit, zu denen grösstenteils die Arbeiten von X. Kostriukova gehören, zeigten die Richtigkeit meiner Ansicht in Betreff der systematischen Bedeutung des Baues der männlichen Gametophyten der Angiospermen (¹²): die generativen Zellen und die aus ihnen entstehenden Spermien unterscheiden sich bei den verschiedenen Vertretern der Angiospermen nicht nur durch ihre Plasmamenge—sie sind entweder reich

oder arm an Plasma,—sondern auch durch die Form der Kerne und eine ganze Reihe feinster zytologischer Merkmale.

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CYTOLOGY

CHROMOSOME STRUCTURE AFTER REMOVAL OF THYMONUCLEIC ACID BY ACTION OF ENZYMES

By S. L. FROLOVA

(Communicated by I. I. Schmalhausen, Member of the Academy, 15. XII. 1940)

The problem of chromosome structure cannot be solved at present without application of chemistry and molecular physics. N. K. Koltzoff was the first to speak of the chromosome as of a giant molecule (1927); later on he developed the same idea in connexion with the modern theory of highly polymeric molecules ([3-6]). Similar ideas are found in articles by Wrinch ([9])and in Frey-Wyssling's book ([8]). The structural framework of the chromosomes is formed of highly polymeric polypeptide molecules with side chains corresponding to genes. Such a giant protein chain molecule or bundle of parallel molecules forms a genonema, according to Koltzoff. The thymonucleic acid without entering the composition of the genonema becomes attached to the side chains of this giant molecule. In other words, it becomes concentrated only on definite parts of the genonema and calls forth an intense staining of the chromomeres with basic stains ([4], [5]). A number of studies on chromosome structure show that at metaphase the chromosome has no less than four genonema, in the salivary gland chromosomes of *Diptera* they are quite numerous.

The continuity of protein framework in the chromosomes is proved by Caspersson ([1]) and by Mazia and Jaeger ([7]). Caspersson showed that the salivary gland chromosomes of *Drosophila* consist of protein segments (intervals between the bands) alternating with segments formed of protein and thymonucleic acid (bands). Mazia and Jaeger stained the continuous protein framework of the salivary gland chromosomes of *Drosophila* blue by the ninhydrin reaction for protein, after removal of the nucleic acid under the action of enzymes—nucleases.

The problem of the structure of chromomeres remained unsolved. According to Koltzoff, the framework of the chromomeres is of protein nature, the thymonucleic acid forming only a sheath, while other investigators believe the skeletal framework of the chromomeres to be formed of nucleic acid (Caspersson, Wrinch, Kiesel). The present note is confined to the elucidation of this problem.

I modified the method of Mazia and Jaeger and used sections of salivary glands, fixed with alcohol and aceto-carmin preparations, instead of live salivary glands. I did not succeed in obtaining an active nuclease solution of spleen juice (after van Herwerden) applied by Mazia and Jaeger. Therefore, I applied pure tissue juice pressed from beef spleen and diluted with

water, in other words, I applied a complex of enzymes. It was assumed that this juice contained all the enzymes known under the term of nuclease: polynucleotidases, splitting polynucleotides into mononucleotides; nucleotidases, splitting off phosphoric acid under formation of nucleosides; and nucleosidases, splitting off nitrogenous bases and thus completing the decomposition of nucleic acids. The activity of the enzymes was ensured by neutralizing the juice (pH = 7) by means of 0.01 and 0.1 *N* NaOH. The preparations were placed into the spleen juice which was transferred into the thermostat at 37°, and after that the Feulgen reaction was applied. The absence of thymonucleic acid was checked by the negative Feulgen reaction. As control I used preparations placed in the boiled juice and preparations not used in this experiment. The latter served as control for the Feulgen reaction.

In order to obtain a negative Feulgen reaction it was usually sufficient to keep the preparations in the juice for about 20—24 hours; a feeble positive

Fig. 1. Photomicrograph of a salivary gland chromosome region of *Chironomus* after the action of enzymes (nucleases) and a negative Feulgen reaction

Feulgen reaction was noticed after 18 hours. No difference was observed whether the preparations were kept in the juice for 24 or 36 hours. Therefore, I kept the preparations in the juice for 36 hours to be confident that the thymonucleic acid was entirely removed. No difference was observed in case the preparations were kept in the juice for several days. After a negative Feulgen reaction the salivary gland chromosomes are so transparent in Canada balsam, that they can scarcely be distinguished, and one frequently has the impression that they are almost entirely destroyed. However, if the same preparation is examined in glycerin-gelatin, the entire structure of the chromosome which was seen before the action of the enzymes is visible (Figs. 1, 2), although they are completely colourless. The separate chromomeres, as well as the bands, are clearly seen, while the genonemata situated between the bands are more clearly visible than before the experiment.

Visual observations, as well as photomicrographs, show no appreciable decrease in the chromomere size in the euchromatin regions of the chromosomes. The heterochromatin regions of the *Drosophila* chromosomes are divided into two groups: some are very compact, similar to those in the autosomes of *D. robusta* and in the *X*-chromosome of *D. hydei*; others show a feeble conjugation between the genonemata which separate easily when the aceto-carmin preparation is made, and make the impression of a structureless granular mass, as in *D. melanogaster* and in the *X*-chromosome of *D. robusta* and *D. funebris*. After the action of enzymes, the chromomeres were found in both types of heterochromatin regions. These chromomeres are

identical with those before the experiment. However, the chromomeres in the heterochromatin regions of *D. robusta* seem to be considerably smaller.

After a negative Feulgen reaction the preparations were stained with a number of basic and acid stains. Borax carmin, toluidin blue, methylene green and haematoxylin gave negative results. When the preparations are kept for a long time in Karazzi's haematoxylin, a diffuse staining of the chromomeres, genonemata and surrounding plasm is observed. Similar

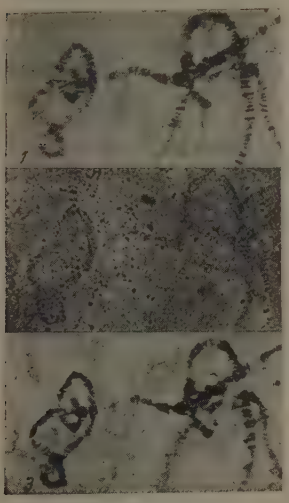

Fig. 2.

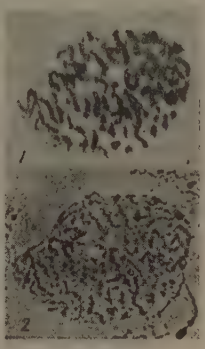

Fig. 3.

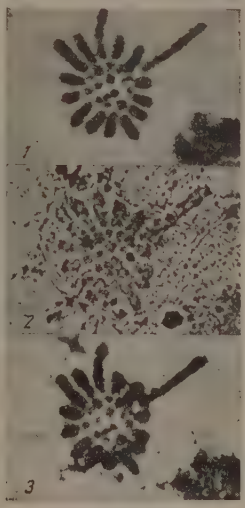

Fig. 4.

Fig. 2. Photomicrograph of a salivary gland chromosome of *Drosophila melanogaster*: *1* — aceto-carmin staining: *2* — after the action of nucleases and of the negative Feulgen reaction; *3* — subsequent staining with acid fuchsin.

Fig. 3. Photomicrograph of a grasshopper spermatocyte: *1* — aceto-carmin staining; *2*—after the action of nucleases and of the negative Feulgen reaction.

Fig. 4. Photomicrograph of a metaphase plate of a grasshopper's spermatogonium: *1* — aceto-carmin staining; *2* — after the action of nucleases and of the negative Feulgen reaction; *3* — subsequent staining with acid fuchsin.

results are produced when acid stains, lichtgrün and acid fuchsin are applied (Figs. 2).

The results obtained were verified on other objects. The salivary gland chromosomes of *Chironomus* gave identical results. The aceto-carmin preparations of the spermatogenesis of the grasshopper showed chromomeres at all maturation stages of the spermatozytes: leptotene, pachytene, diplotene, diakinesis (Fig. 3).

The metaphase chromosomes in the spermatogonia of the grasshopper (Fig. 4) and in the root cells of *Vicia faba* do not diminish under the action of enzymes; spiral chromonemata which were visible in the aceto-carmin preparations are preserved after the action of the enzyme. At this stage the chromomeres cannot be visible, since the shortening of the chromosomes is caused by the spiralization of the genonemata: this causes the close contact of the chromomeres giving the impression of an entire thick thread.

As a result of the above, the question naturally arises first of all, whether the thymonucleic acid is entirely removed and whether the remaining structural frameworks are of entirely protein nature. No ultimate solution of this question can be obtained until subsequent experiments with the application of optical methods have been carried out. At the present time the nin-

hydrin reaction of protein (Mazia and Jaeger), the negative Feulgen reaction with and without HCl hydrolysis, and the fact of non-staining with basic stains speak for an entire splitting of the nucleic acid. Even in case remnants of nucleic acid were preserved, neither the latter nor the molecules of the plasm surrounding the genonema would be capable of constructing the chromomere. I believe that the genonema consists of a large bundle of parallel protein chain molecules; according to Wrinch, such a bundle may have a diameter of 100 Å and more. In this case the chromomeres are formed of similar parallel bundles of polypeptide side chains. The molecules of the thymonucleic acid of any shape (elongated, cyclized) become attached to the protein on the surface of the chromomere as well as within it; the splitting off of these molecules and their subsequent disintegration can, therefore, cause no essential changes in the size or shape of the chromomeres, whose protein skeletal framework has previously undergone fixation with acetic acid and alcohol and has not been subjected to the action of enzymes. The considerable reduction of the chromomere size in the inert regions of the chromosomes shows that here the molecules of the thymonucleic acid combine with the protein mainly on the surface of the chromomere.

My experiments on the structure of the chromosomes after the action of enzymes are in progress. However, I find it possible to formulate my first conclusion on the subject. I believe that my data refute the point of view that thymonucleic acid forms the skeletal framework of the chromomeres whose loops contain protein ([1]) and confirm the opposite theoretical conception ([3–6]).

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CYTOLOGY

ON THE PELLICULAR LAYER OF THE CYTOPLASM OF THE GENERATIVE CELL OF *CONVALLARIA MAJALIS* L.

By X. J. KOSTRIUKOVA

(Communicated by A. A. Richter, Member of the Academy, 9. XI. 1940)

Investigating in vivo the generative cell in the artificially germinated pollen tubes, I could state that in *Lilium martagon* L. as well as in other *Liliaceae* (*Tulipa Gesneriana* L., *Paris quadrifolia* L.) one can observe sometimes very clearly a pellicular outer layer of the cytoplasm of the generative cell ([1]).

There are two opposite opinions in literature as to the existence of the pellicular layer of the cytoplasm of the generative cell in angiosperms: some authors, basing on the investigation of the fixed material point out that they failed to observe the pellicular layer of the investigated material ([2-4]), others note the existence of a membrane wall (Wand, Wandschicht), or a pellicular layer (Hautschicht) bordering the body of the generative cell ([5-13]). Most of the authors, describing the development of the generative cell, simply omit the question as to the formation of a pellicular layer in the cytoplasm.

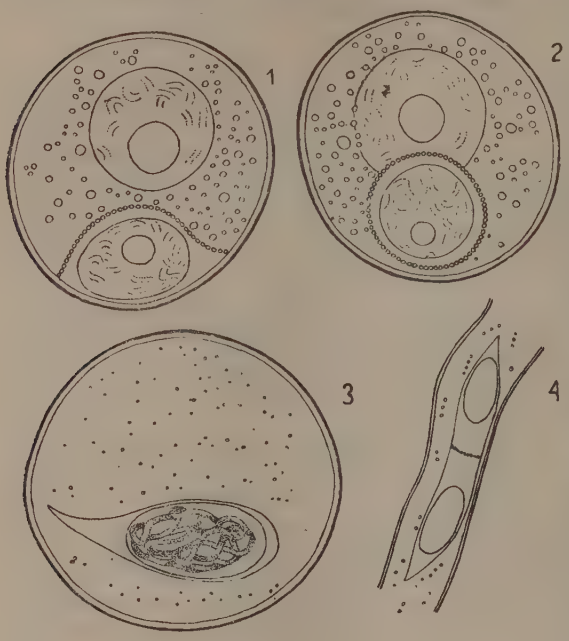
The above compelled me to investigate the earlier stages of development of the generative cell which proceed as early as in the pollen grain. For my investigations I have chosen *Convallaria majalis* whose pollen grains present a particularly convenient material for examination in vivo, because of the ideal transparency of the exine, which does not prevent detecting the most minute details in the structure of vegetative and generative cells. The pollen grains were examined in paraffin oil without any staining.

The earliest stage I could observe was that of the generative cell at the wall (Fig. 1). At this stage the pollen grains of *C. majalis* are quite round. The cytoplasm of the vegetative cell is filled up with rather large granules somewhat differing as to their size*. Though the granules are very abundant, the vegetative nucleus and the generative cell may be seen very well. The vegetative nucleus is perfectly round. Its outlines are very well defined at this stage of development; it is not so transparent or so poor in chromatin as the nuclei I have been observing in the pollen grains of other plants ([1,15]) at later stages of development. It shows indefinite structures and a large nucleolus.

The generative cell which lies at the wall of the pollen grain shows the shape of a biconvex lens. The round generative nucleus is very large. Its structure is characteristic for resting nuclei rich in chromatin ([16]): it contains

* It should be noted that the size of the granules varies somewhat in different flower shoots.

thread-like highly refractive structures. The nucleolus, which is very much smaller than that of the vegetative nucleus, may be well seen. The cytoplasm of the generative cell is quite transparent and structureless. It surrounds the nucleus as a thin layer, and is clearly seen only at both poles of the stretched lens-shaped cell. The outer surface of the generative cytoplasm, contiguous to the vegetative one, is edged with a distinct strongly refractive thin border-edge (Fig. 1)*. In the optical section, under favourable conditions this layer seems to be formed by one row of small very strongly refractive granules, or droplets of the same size and regular spherical shape (Fig. 2). This outer layer evidently corresponds to the formation named «uhrglasförmige Wand» by former authors ([7], [12]).



Figs. 1—4.

One can follow on the pollen grains, taken from the same anther, all the process of separation of the generative cell from the wall of the pollen grain and its incorporation into the cytoplasm of the vegetative cell.

Further the generative cell grows round. The cytoplasm surrounds the nucleus with an equal layer so thin, that its presence could be found out in vivo, only because the nucleus very rich in chromatin appears considerably darker than the structureless cytoplasm. The granular structure of the outer layer of the generative cytoplasm is seen very clearly at this stage, especially in cases when the generative cell lies under the wall of the pollen grain turned to the objective of the microscope. Because of the strong light refractivity of the granules, the outer layer can be seen even at low magnifications of the microscope. Then in optical section the generative cell resembles very much a bubble of air by its sharp contours (Fig. 2).

At further stages of development one can observe how the shape of the generative cell begins to change. Its body elongates somewhat, forming a sharp process first at one, then at another side. The granular layer bordering the body of the generative cell is not so sharp, its refractivity decreases considerably. One can still see separate granules.

Later on the generative cell stretches still more. Its nucleus elongates considerably, passing to the stage of prophase. The strongly refractive curves of chromatin threads may be clearly seen. At this stage of development I failed to distinguish the outer layer of the cytoplasm of the generative cell which is so characteristic of the early stages of its development. However,

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* All figures are made with Abbe's drawing apparatus with magnification: ob. apochr. 90 Zeiss × oc. 10, except Fig. 4—ob. apochr. 90 Zeiss × comp. oc. 6.

the boundary between the vegetative and the generative cytoplasms is still seen. The vegetative cytoplasm has no inclusions and appears quite transparent. The vegetative nucleus is usually not to be seen anymore. It should be supposed that these colourless transparent formations have now the same degree of refractivity owing to which they are not distinguishable in vivo (Fig. 3) *. In riper pollen grains which assume a very graceful elongated shape**, except rare cases, one cannot distinguish even the body of the generative cell. The contents of the pollen grain are quite transparent. Sometimes one can find out the location of the generative cell, following the curves of the chromatin threads which are at this stage of development the most distinct formations in the pollen grain in *C. majalis*.

What then is the outer strongly refractive layer of the generative cell? It is not a hard cell wall, does not react with cellulose, which has been pointed out as far back as by Elfving and Strasburger ([7], [12]). This layer is supple, its shape changes in accordance with the changes that occur in the shape of the generative cell. As to its qualities (high degree of refractivity, absence of stain reactions, which distinguish it from the cytoplasm), it resembles most of all a formation which is called the pellicular or membranous layer of the cytoplasm ([18]). The evolution of this layer proceeds in connexion with that of the generative cell. The degree of its refractivity gradually decreases and in riper pollen grains it is indistinguishable from the rest of the generative cytoplasm. However, it can be seen again in the pollen tube. Particularly clear-cut is the pellicular layer between the sperm cells which form at the division of the generative cell. In *C. majalis* I did not succeed to discover any granular structure, yet this may possibly be due to the rapid current of the cytoplasm of the pollen tube. However, in other plants of the same subfamily *Asparangoideae* (e. g. *Asparagus officinalis* L., Fig. 4) I could see quite distinctly the granular character of a light strip—a partition which forms between the sperm cells and separates them.

I do not doubt that the pellicular outer layer of the cytoplasm which I discovered in *C. majalis* is not a formation peculiar to it alone. It should be assumed that in all the angiosperms the outer layer of the generative cell, through which proceeds the metabolism with the vegetative cell, has particular qualities which are manifest in a high degree of refractivity at the early stages of development of this layer in the generative cell and sperm cells, at stages of their most intense metabolism.

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* The right end of the generative cell does not lie in the same optical section and is not shown in the figure.

** The round shape of the ripe pollen grain of *C. majalis* described by Trankowsky ([17]) is an artefact due to water fixative.

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PHYTOPATHOLOGY

PROTEOLYTIC ACTIVITY OF A PREPARATION FROM TOBACCO MOSAIC VIRUS

By M. N. VOROBIEVA

(Communicated by A. N. Bach, Member of the Academy, 3. X. 1940)

The unsettled state of our knowledge concerning the physiological rôle of the virus protein in the life of plants affected with tobacco mosaic, calls for further research. So the present paper is concerned with the proteolytic activity in such plants,—a question, to which only one work appears to have been devoted as yet (1). That single work was not free from several drawbacks, among which was the use of peptone and gelatine as substratum, whereby the specific nature of vegetable proteolytic enzymes, reported by different authors, was not taken into account. Neither did this point receive due attention in a work by Rischkov and Sukhov (2), whose substratum was also a protein alien from the virus.

I therefore have studied the proteolytic activity using as substratum the tissue of the plants, on which the virus is liable to develop.

A preparation of tobacco mosaic crystalloid virus, obtained according to Rischkov and Gromyko (2), was dialyzed until a negative reaction for NH_3 with the reagent of Nessler. It is in this state that the virus preparation was used in the tests. As experimental material were chosen the seedlings of tomato, variety King Humbert, which were brought to germination in distilled water.

The first test was run with seven-day-old seedlings, while in all the others the seedlings had just started growing. After their surface water was wiped out with filter paper, samples were taken from an equal number of plants for humidity estimations and for direct investigation of enzymic activity.

The weighed out plants were thoroughly ground in the mortar, from which the contents were transferred quantitatively into Ehrlenmeyer flasks by an acetate buffer solution with pH=5. After boiling during 5 min. and cooling the experimental flasks were added with 1 cm^3 of virus, while the controls were added with as much distilled water. As an antiseptic toluol was employed in excess. Closed with rubber stoppers, the flasks were set in a thermostat at 32° C for 106 hours. In a cubic centimeter such a preparation of virus contained 0.0147 mg of non-protein nitrogen and 0.38 mg of protein nitrogen. Later on these quantities were subtracted from the experimental data. Standing in the thermostat was followed by precipitation of protein according to Barnstein. In estimating nitrogen the micromethod of Kjeldahl was employed. The data obtained in one of the tests are summarized in Table 1.

Table 1

Protein Breakdown in Tomato in Response to Crystalline Virus of Tobacco Mosaic (seven-day-old plants, exposure 106 hrs).

Experimental plan	Protein N		Ratio of protein N to the control put at 100
	mgs in dry matter	% by weight in dry matter	
Control, 5 min. boiling	1.8508	2.883	100.00
Boiling + 1 cm³ of virus	1.5652	2.401	83.20
Autolysis	1.5988	2.687	93.21

It will be obvious from Table 1, that in a vegetable substratum, whose proteolytic enzymes have been killed previously, the hydrolysis of proteins still arises after addition of 1 cm³ of tobacco mosaic virus. These results were confirmed by a series of experiments with younger seedlings.

Simultaneously we made an attempt to establish whether the proteolytic activity of the virus might be stimulated by some promoter. We tried potassium cyanide in a 0.31% concentration. Its stimulative influence on proteolytic activity in plants is well known. In Table 2 are given the results of this test.

Table 2

Protein Breakdown in Tomato by Tobacco Mosaic Virus Stimulated with Potassium Cyanide (three-day-seedlings, exposure 106 h.)

Experimental plan	Non-protein N		Protein N		Total N, % by weight in dry matter	Protein ratio to the control put at 100
	mgs in dry matter	% by weight in dry matter	mgs in dry matter	% by weight in dry matter		
Control, 5 min. boiling . .	0.2856	0.4776	2.1786	3.643	4.1207	100.00
5 min. boiling + 1 cm³ of virus	0.5768	1.1028	—	—	—	—
5 min. boiling + 0.5 cm³ of activated virus	1.2684	2.5521	0.9118	1.6135	4.1656	44.30

The results obtained can be explained in two ways: either it is by the virus itself that the proteins are hydrolyzed in tomato seedlings, or the virus preparation we used served only to adsorb on its surface the proteolytic enzymes of the plants, and it is this action that we have observed in our tests.

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HISTOLOGIE

DIE WIRKUNG DES CHININS AUF DAS WACHSTUM DER ELEMENTE DES EMBRYONALEN HERZENS AUSSERHALB DES ORGANISMUS

Von L. S. SUTULOV

(Eingegangen am 1. 9. 1941. Original russisch. Manuskript vor Abdruck an d. Red. XI. 1941)

Die Wirkung verschiedener Arzneimittel auf das Wachstum von Herzelementen ist von vielen Forschern untersucht worden [1–5], und diese Aufgabe ist, aber es muss bei weitem noch eine Reihe von prinzipiellen Momenten über die Natur der beobachteten Effekte, so Lewis Gemeil (6), Parker (7), Fischer (8) und andere, die Elemente, wurden in den Kulturmedien die Wachstumsstoffe entdeckten, als wesentliche Bedingung für das Wachstum der Herzelemente ausserhalb des Organismus gefunden [Ebeling, Levi, Ephrussi u. a.].

In vorliegender Mitteilung werden Versuche beschrieben, die bei der Untersuchung der Gesetzmässigkeiten der Einwirkung von Chinin auf Herzelemente embryonaler Hühnerembryonen entdeckten Veränderungen gemacht wurden sind. Diese Objekte haben die zahlreiche vergleichende Versuche gedient und viele Verfasser sind der Meinung, dass diese Objekte die beständigsten Resultate ergeben.

Die Versuchen dienten Herzgewebsstücke des 4-tägigen und 5-tägigen Hühnerembryos. Die Explantate wurden nach der Methode des hängenden Tropfens angefertigt. Die Kulturen wurden alle bei Zimmer- und manchen Thermostattemperatur angelegt. In einigen Fällen geschah die Umlegung nach Carrel's Methode—durch Herzmuskelzellen. Chinin wurde in Gestalt seines salzsauren Salzes gebraucht.

Die Auswertung der Untersuchungen erfolgte mehrere Tage durch bei der Beobachtung oder nach vorgenommenen Fixierungen und Färbungen. Das angefertigte Präparat wurde in zwei Portionen geteilt. Die eine wurde in Ringerlösung ohne Chinin aufbewahrt, die andere mit der Kontrollkultur gleich, wurde mit Ringerlösung gleicher, aber Chinin versetzt. Im Versuch, wie bei der Kontrolle, wurden zu bestimmten Zeitabschnitten, und zwar in einer bestimmten Menge Plasma wurde eine gewisse Menge der Ringerlösung angesetzt (gleichzeitige Ringerlösung im Versuche und gleiche Ringerlösung in der Kontrolle). Aus den Kulturen wurden die ersten Präparate angefertigt, die anderen nach Färbung mit Hämatoxylin, Formolfixierung und Färbung mit Hämatoxylin nach Carrel.

Schon im Laufe des ersten Tages zeigt sich bei Beginn des Wachstums an den explantierten Herzgewebsstücken bereits bemerkbare Abweichungen. Verschiedenartige Elemente eines Explantats. Von den zweiten Tag an verschwinden sich die Kulturen voneinander. In den ersten Tagen des Wachstums der Herzgewebsstücke gibt, und diese Wachstum wurde besonders stark

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HELMINTHOLOGY

ON THE REARRANGEMENT OF THE TAXONOMY OF NEMATODES OF THE *ACUARIIDAE* AND *ANCYRACANTHIDAE* FAMILIES

By K. I. SKRJABIN, Member of the Academy

14 years have elapsed since the appearance of Yorke and Maplestone's classical work «The Nematode Parasites of Vertebrates», which is the first attempt to give a consistent summary of the taxonomy of nematode parasites of vertebrates. Since then new investigations made by Soviet and foreign (mainly American) helminthologists have accumulated, which make it necessary to revise the principle of the English authors and suggest a number of important corrections.

In the present paper I will discuss only two nematode families of the *Spirurata* suborder: the *Acuariidae* and *Ancyracanthidae* whose taxonomy should not only be revised, but entirely rearranged.

As it is known, Seurat founded in 1913 the *Acuariidae* family, which includes the subfamily *Acuariinae* Railliet, Henry et Sisoff, and the subfamily *Physalopterinae* Raill., 1893. Within the next years the number of genera of this family went on increasing rapidly. Skrjabin founded in 1915 the genus *Sciadiocara*, in 1916 the genus *Seuratia*; Seurat described in 1918 the genus *Cheuvreuxia* and in 1919 the genus *Rusguniella*. Baylis (1921) described the genus *Parabronema*, Vassilkova (1926)—*Antennocara*, Gilbert (1927)—*Syncuaria*, Wehr (1931—1933)—*Aviculariella* and *Pectinospirura*, Gushanskaja (1937)—*Stellobronema*, Kurashvili (1940)—*Skrjabinocara*, Sobolev (1939—40)—*Sexansocara* and *Skrjabinoclava*. Gushanskaja included her genus *Stellobronema* in the subfamily *Schistorophinae* Travassos 1918, separating the latter from the *Ancyracanthidae* family, and added the *Schistorophinae* to the *Acuariidae* family. Sobolev (1940) isolated from the *Acuariidae* the genera *Echinuria*, *Seuratia*, *Skrjabinocerca* and *Skrjabinoclava*, forming for them a new subfamily *Echinuriinae* whose diagnostic character consists in chitinous spines along the body, other specific *Acuariidae* characters being preserved.

Thus, thanks to investigations of Gushanskaja and Sobolev, by the beginning of 1940 the *Acuariidae* family numbered 3 subfamilies: 1) *Acuariinae* Raill., Henry et Sisoff, 1912; 2) *Schistorophinae* Travassos, 1918, and 3) *Echinuriinae* Sobolew, 1940.

Let us try and define the dominant anatomical morphological characters of each genus of the above subfamilies in order to make a better analysis of their respective taxonomic position.

It seems unquestionable to me that the presence of such a specific organ as the cordons shown by the representatives of a number of genera, should be regarded as a character of family significance. Consequently, I shall class among the *Acuariidae* only such *Spirurata* representatives which possess

the said cordons. The presence of two labia, of unequal spicules and of four pairs of preanal papillae is correlated with the above character. The combination of these features characterizes the diagnosis of the *Acuariidae* Seurat, 1913. However, this family includes genera of a rather varying type. In some of them the presence of elongated cordons is accompanied with the presence of chitinous spines distributed over their bodies in a certain order. Such genera should be safely referred to the subfamily *Echinuriinae* Sobolew, 1940, in contrast with the genera of the subfamily *Acuariinae* Raill., Henry et Sisoff, 1912, whose representatives bear cordons, directed backwards from the oral apparatus (which may either curl back, or else may not curl and form no anastomoses), but whoce cuticle has no chitinous armament.

The genus *Streptocara* which by Yorke and Maplestone has been quite arbitrarily referred to the family *Physolopteridae* Leiper, 1908, stands somewhat apart. In fact, the *Streptocara* show all the *Acuariidae* characters: they possess two labia, four pairs of preanal papillae, unequal spicules and dentate cervical papillae at the posterior part of the body, completely corresponding to those of the *Synhimantus* and of some other *Cosmocephalus*. On the other hand, the *Streptocara* have no well expressed cordons directed backwards as it is the case in *Acuariinae*, and their cuticle bears no chitinous armament like that of *Echinuriinae*. However, a thorough examination of the structure of the anterior part of the *Streptocara* reveals the presence of an uneven or sometimes dentate necklace encircling the head of the nematode. This necklace is undoubtedly homologous to the «cervical cordons» of the *Acuariidae*. This is why I find it necessary to isolate the genus *Streptocara* from the *Physalopteridae* and to include it in the *Acuariidae*, founding a new subfamily *Streptocarinae* nov. subfam.

Analyzing the structure of the genus *Ancyracanthus* Dies, 1838, I arrive at the conclusion that since the appendices of the anterior part of the body bearing two labia are to be qualified as homologues of the cervical cordons of the *Acuariidae*, the said genus should be excluded from the *Ancyracanthidae* family and considered also as a representative of the *Acuariidae* family. However, taking into consideration such structural peculiarities as the presence of two equal spicules and of two pairs of preanal papillae, instead of four pairs, which is characteristic of other *Acuariidae*, as well as the fact that they are reptilia (turtle) rather than bird parasites, I find it expedient to consider this genus as a representative of a separate subfamily *Ancyracanthinae* Yorke et Maplestone, 1926, entering the *Acuariidae* family. The family *Ancyracanthidae* Railliet, 1916, can thus be considered as good as cancelled.

Now, let us analyze the position in the *Spirurata* system of such genera which by Yorke and Maplestone were united in 1926 in the subfamily *Schistorophinae* Travassos, 1918. All of them are characterized by the absence of the cervical cordons and the chitinous armament of the cuticle. At the same time the anterior part of their body is adorned with a complex of outgrowths, spines and swellings of the cuticle. Besides these characters, most of the genera of this subfamily are characterized by a great number of preanal papillae (more than 4 pairs). These morphological anatomical peculiarities make me conclude that the genera of the *Schistorophinae* subfamily cannot be referred to the *Acuariidae* family. As I was forced to cancel the *Ancyracanthidae* family which, according to Yorke and Maplestone, included the subfamily *Schistorophinae*, it becomes necessary to separate the latter as a new family, *Schistorophidae* n. fam.

Yorke and Maplestone referred to the subfamily *Schistorophinae* 7 genera: *Schistorophus* Raill., 1916; *Sciadiocara* Skrj., 1915; *Ancyracanthopsis* Dies., 1861; *Histiocephalus* Dies., 1851; *Torquatella* Yorke et Maplest., 1926; *Serticeps* Raill., 1916; *Viguiera* Seurat, 1913.

I believe that *Schistorophus*, *Sciadiocara* and *Ancyracanthopsis* form a special taxonomic category, characterized by four appendices on the head, a pair on each of the two lips. I feel inclined to isolate these genera into a separate subfamily: *Schistorophinae* Travassos, 1918.

On the other hand, *Serticeps* and *Torquatella* which differ by the structure of the head region from the three genera described above, have one common feature, namely, a cuticular collar of complicated structure behind the lips consisting of peculiar outgrowths and basreliefs. I find it necessary to unite both these genera into a separate subfamily *Serticepsinae* nov. subf. And thereby I feel in no way embarrassed by the fact that in the case of *Torquatella* there are described two labia, while their number in *Serticeps* is said to be six; I believe that the *Serticeps* has two lateral lips, but each of them is subdivided into three lobes, as in many *Spirurata*, which led some investigators of the 19th century (Molin, Drasche) to the erroneous conclusion that the *Serticeps* had six labia instead of two.

As to the taxonomic position of the genus *Viguiera*, the said genus, judging by the structure of the caudal region, corresponds entirely to the diagnosis of the *Schistorophidae*; however, owing to the specific structure of the head region (absence of lips, presence of paraoral disk in the form of a ring-like swelling), it can be referred neither to the *Schistorophinae*, nor the *Serticepsinae*. Therefore, though including the genus *Viguiera* in the family *Schistorophidae*, I class it as a special subfamily *Viguierinae* nov. subf.

Judging by the structure of the head region (peculiar outgrowths), the peculiar genus *Histiocephalus* Dies, 1851, tends toward the *Schistorophidae*, differing from the representatives of the latter in the presence of four pairs of preanal papillae, which connects this genus with the *Acuariidae*, on account of which it can be included neither in the *Acuariidae* (absence of cordons), nor the *Schistorophidae* (different character of papillae on the tail in males). Consequently, I am forced to regard this genus as a representative of a separate new family, *Histiocephalidae* nov. fam. I am inclined to class among this family two other genera which, notwithstanding the similarity of their organization, have been referred by Yorke et Maplestone to two different families. I mean the *Parabronema* Baylis, 1921 (*Acuariidae* family), and the *Squamonema* Thiel, 1925 (*Spiruridae* family, subfamily *Gongylonematinae*). The head region of the representatives of these genera bears dorsal and ventral cuticular scutellae dividing in the posterior part into 4—6 lobes. Besides, as in the case of *Histiocephalus*, the structure of the male's caudal region is complete (four pairs of preanal papillae). I separate both these genera into a new subfamily *Parabronematinae* nov. subf.

To the same family I must add the following interesting genera: *Stellocaronema* Gilbert, 1930, and *Stellobronema* Guschanskaja, 1937, whose structure of the head region is much like that of the *Schistorophidae*, whereas the male's caudal part is characterized by the presence of four pairs of preanal papillae and unequal spicules. These genera should also form a separate subfamily *Stellocaronematinae* nov. subf.

Thus, I find it necessary to split the family *Histiocephalidae* into 3 subfamilies: 1) *Histiocephalinae* n. subf. with a genus *Histiocephalus*, 2) *Parabronematinae* with the genera *Parabronema* and *Squamonema*, and 3) *Stellocaronematinae* with genera *Stellobronema* and *Stellocaronema*.

These subfamilies will differ from each other both in the structure of their head region (specific ornamentations in representatives of each subfamily) and the character of the male's caudal region (equal or unequal spicules).

To sum up, my rearrangement of the taxonomy of the nematodes *Acuariidae* and *Ancyracanthidae* is reduced to the following:

A) Family *Acuariidae* Seurat, 1913

I. Subfamily *Acuariinae* Raill., Henry et Sisoff, 1912, with genera: 1) *Acuaria* Bremser, 1811; 2) *Cheilospirura* Dies, 1861; 3) *Dispharynx* Raill., Henry et Sisoff, 1912; 4) *Synhimantus* Raill., Henry et Sisoff, 1912; 5) *Chevreuxia* Seurat, 1918; 6) *Rusguniella* Seurat, 1919; 7) *Cosmocephalus* Molin, 1858; 8) *Antennocara* Wassilkowa, 1926; 9) *Syncuaria* Gilbert, 1927; 10) *Sexansocara* Sobolew et Soudarikow, 1939; 11) *Pectinospirura* Wehr, 1933; 12) *Aviculariella* Wehr, 1913; 13) *Skrjabinocerca* Schikhobalowa, 1930.

II. Subfamily *Echinuriinae* Sobolew, 1941, with genera: 1) *Echinuria* Solowiew, 1912; 2) *Seuratia* Skrjabin, 1916; 3) *Skrjabinoclava* Sobolew, 1940; 4) *Skrjabinocara* Kurashwili, 1940.

III. Subfamily *Streptocarinae* Skrj., 1940, with genus *Streptocara* Raill., Henry et Sisoff, 1912.

IV. Subfamily *Ancyracanthinae* Yorke et Maplestone, 1926, with genus *Ancyracanthus* Dies, 1838.

B) Family *Schistorophidae* Skrj., 1941

I. Subfamily *Schistorophinae* Travassos, 1918, with genera: 1) *Schistorophus* Raill., 1916; 2) *Sciadiocara* Skrj., 1915; 3) *Ancyracanthopsis* Dies., 1861.

II. Subfamily *Serticepsinae* Skrj., 1941; with genera: 1) *Serticeps* Raill., 1916; 2) *Torquatella* Yorke et Maplestone, 1926.

III. Subfamily *Viguierinae* Skrj., 1941, with genus *Viguiera* Seurat, 1913.

C) Family *Histiocephalidae* Skrj., 1941

I. Subfamily *Histiocephalinae* Skrj., 1941, with single genus *Histiocephalus* Dies., 1851.

II. Subfamily *Parabronematinae* Skrj., 1941, with genera: 1) *Parabronema* Baylis, 1921; 2) *Squamonema* Thiel, 1925.

III. Subfamily *Stellocaronematinae* Skrj., 1941, with genera: 1) *Stellocaronema* Gilbert, 1931; 2) *Stellobronema* Guschanskaja, 1937.

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HELMINTHOLOGY

CONTRIBUTION TO THE BIOLOGY OF *MONIEZIA EXPANSA* (RUDOLPHI, 1810), A TAPEWORM PARASITIC IN SHEEP AND GOATS

By V. A. POTEMKINA

(Communicated by K. I. Skrjabin, Member of the Academy, 12. XI. 1940)

In spite of all the attempts made by the students to retrace in detail the cycle of development of *Moniezia expansa* and other related forms, the biology of these cestodes remained obscure until 1937 when Stunkard reported *M. expansa* cysticercoids from the body of the mites *Galumna*, infected experimentally with mature eggs of the cestoid in question. The fact that mites *Galumnidae* (*Oribatei*) act as intermediate hosts for *M. expansa* has been corroborated by recent investigations of Stoll ([2]), Stunkard ([3,6]) and Krull ([4–5]).

This discovery is most valuable both for helminthology and practical veterinary. On the one hand, it gives a new trend to the development of prophylactic measures and methods for combating monieziosis of sheep and goats in our farms; on the other hand, it indicates the proper method of investigation of the biological cycle of cestoids belonging to the *Anoplocephalidae* family.

The articles of the American authors cited above, which are merely preliminary notes, point out that in the USA the intermediate host for *M. expansa* is the mite *Galumna emarginata* Bonus, which is unknown in the USSR. Consequently, our first task was to find out the intermediate hosts responsible for spreading monieziosis among the sheep in this country, and to investigate their habitats, biology, etc.

First of all it was necessary to work out a method for collecting mites of certain groups which are known to live on the soil surface, in the moss, wood bedding, decaying leaves, etc. We collected the substratum inhabited by *Galumna*—mites near Moscow, in places where cattle did not pasture. In order to obtain mites in necessary quantities, we used large metal funnels about 73 cm in diameter, the diameter of the tip being about 3—4 cm. The latter was put into a porcelain glass. The mouth of the funnel was covered with gauze upon which 0.5 kg of soil substratum was placed. The apparatus charged in this manner was exposed to the light of a common electric bulb. The mites fled from the substratum, dropped to the bottom of the funnel and into the porcelain glass, where they were collected. Since the 1st of May till the 1st of November 1940 we collected by this method 30000 oribatidae mites belonging to different species. For further experiments the collected mites were kept in closed small glass vessels with perforated lids, sometimes the vessels were covered with mill gauze or marquizette. The vessels were placed into a moist chamber and kept in a thermostat at 19—21°. Some mites were kept

in test-tubes 2—3 cm large. Some water was poured into the test-tubes to cover the bottom; they were plugged with cotton upon which was placed some filter paper. Soil mixed with moss, decaying wood, dry leaves was used as bedding and food. The mites were infested by feeding them with eggs from mature proglottids of *Moniezia expansa* obtained from the intestine of sheep butchered at the slaughter house. A total of 12000 oribatidae mites were infested during the period from July 30 to November 1.

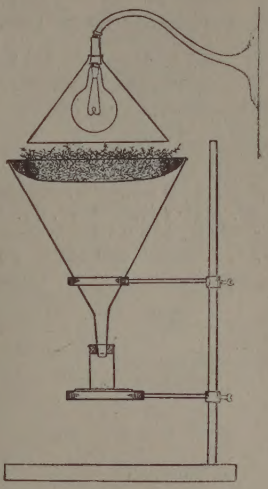

Fig. 1. Apparatus for collecting oribatidae mites.

In order to find out whether the larval form of *Moniezia* develops in the mite's body, we dissected at different periods about 1000 oribatides, among which prevailed representatives of the *Galumnidae* family.

We found that the mites devoured the *Moniezia* eggs greedily, and already within 24 hrs we found in their intestine either oncospheres surrounded with the pear-like apparatus or empty shells of *Moniezia* eggs. 48 hours after infestation mobile oncospheres were already present in the coelom. The oncospheres found 6 days after feeding the mites with *Moniezia* eggs, showed 6 hooks; their diameter was up to 0.029—0.023 mm. By the 21st day after invasion the coelom of the mites contained mature spherical larvae supplied with an appendix at one of their poles. The remaining hooks were shifted towards the appendix, the diameter of the larvae was 0.075×0.068 mm.

At the age of about 30 days the spherical larvae, 0.15×0.16 mm large and filled up by a granular substance, possessed a small cavity; their caudal appendix reached 0.14 mm. Embryonic hooks were scattered in disorder along the tail. At the age of about 40 days the larvae were stretched, 0.32 mm long and 0.14 mm wide. In the centre of the broad (head) region, several calciferous bodies were present and four suckers were seen to form.

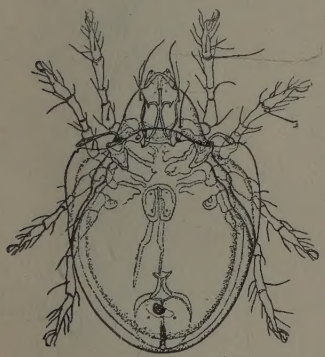

Fig. 2. Mite *Scheloribates laevigatus*, the intermediate host for *Moniezia expansa* (original).

Larvae aged 40—50 days, developing into cysticercoids, reached 0.16 mm in diameter; they were provided with a caudal appendix, four suckers and showed well expressed membranes. Calciferous bodies were spread over the surface of the body. 70—77 days after the invasion the bodies of many mites harboured cysticercoids with a mobile scolex. The capsule was solid and the calciferous bodies were present in large number both in the centre and on the periphery. The length of the cercomere reached about 0.24 mm.

It should be noted that in some cases we observed several underdeveloped *Moniezia expansa* larvae in one mite. Thus, we have succeeded to find out that in this country two mite species act as intermediate hosts for *M. expansa*, and these are: *Galumna obvius* Berl. (belonging to the *Galumnidae* family) and *Scheloribates laevigatus* (*Ceratozetidae* family).

Besides, the above mite species were infested with eggs of cestodes *Moniezia benedeni* (Moniez, 1879) and *Thysaniezia ovilla* (Rivolta, 1878) Skrjabin,

1926. Dissection of mites revealed the presence of cystocercoids of both tapeworm species.

Observations on the development of these three anoplocephalids in oribatidae mites are in progress. The species of the mites which served for our experiments have been determined by A. A. Zakhvatkin.

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ERRATUM

Vol. XXIX, No. 8 — 9, p. 610, article by A. I. Zujtin: the title should read: «Influence of the Change from the Laboratory Complex of Developmental Conditions to the Natural One on the Mutation Rate in *Drosophila melanogaster*».

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